INFINITE SERIES in HEURISTIC

solidity, coherence and elegance of the infinite series in heuristic maths

Andrea Sorato

0) Index

For many methods we will see the types of series:

- Alternating stable

- Divergent with constant terms

- Divergent with no constant terms and no pattern

The patterned series will be described in a separate chapter due to their similarity with all the types of series previously listed.

1) Introduction

1a) General introduction

If you find mistakes, typos or unclear explanations send an e-mail to:

andrea.thesimplepub@gmail.com

Check the following page before starting reading:

thesimplepub.com/bookerrors

or **thesimplepub.wordpress.com/bookerrors**

for the list of typos and mistakes found after the publication of the book, and eventual better explanations of topics not explained well in this book.

My socials links and contact are at the end of the book.

- Andrea Sorato

This book is a translation made by me of my book "SERIE INFINITE in EURISTICA solidità, coerenza ed eleganza delle serie infinite in euristica matematica" in italian language.

I'm not very good in English, and at the moment I cannot afford a professional translation; thus, there will be a lot of mistakes, and sometimes the mathematical terms will be used not correctly.

If you want to have a better explanation of a topic, or you want to report a mistake feel free to send me a message (contacts at the end of the book, some here above).

The topic is: **infinite series in heuristic and their solidity, coherence and elegance.**

Thus, the book will not describe mathematically the topic: heuristic maths approaches the problems **ignoring on purpose some of the math rules,** using intuition and not rigorous methods, to get results which seems valid and which will have to study better in math.

Results might be not right, and the book is a description of procedures I used, many of them have not been verified by mathematicians. However, a lot of things I treat as hypothesis have already been studied by other mathematicians. I studied this topic mostly as autodidact.

Methods and results I will describe can be linked to more rigorous maths methods and evaluations which I will not cover in this book and have already been used to study this topic by other people.

Recently I had the opportunity to look to some papers about this topic. I noticed that some results I noticed were in both my paper and the papers of the mathematicians. This mean I've done a good job, or part of a good job.

In this book we will focus on the heuristic methods, their results and questions.

Moreover, we will use like a model approach: like in physics or in science: a model is created, while it works it's ok, if not, it will be adjusted.

Sometimes there will be some auto references.

Often heuristic methods have been criticized and reported as contradictory: sometimes some supposed incoherence and contradiction of heuristic methods have been used as a fact to show heuristic approaches in infinite series are not valid.

Nevertheless,

heuristic methods for infinite series and associated results seem solid, coherent ed elegant.

Although, these methods look like sometimes (and until now) not very efficient.

The book starts from math and heuristic consideration already made by other mathematicians and researchers.

In this book graphs are not precise, because they are made through a simple drawing software, and not a mathematic software.

Underlined parts of text indicate an interesting topic to study, still unresolved from me o which I don’t know or that has not been written in this book for brevity. Moreover in the 12^{th} chapter there will be a brief description of some of the unresolved or still open, and interesting problems.

The book might be not written in a compatible font for maths expression, which might be not well aligned.

1b) Introduction to series

A series is, simply put, a sum of numbers.

es.: 1+2+3+4+5+6+7+8+9+10+11+12

Often, for brevity, we can use dots to avoid writing all numbers:

es.: 1+2+3+4+...+12

We can use variables in the place of numbers too:

a+b+c+d+e+f+g

or only one variable with the index as subscript:

$x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8$

In a series with a finite quantity of terms, we indicate often with N, n or sometimes with # the numbers of terms summed:

example: 1+2+3+4

N=4 or n=4 (or #=4)

Partial sums

A partial sum is the sum of the first n terms, where n is indicated.

Example:

S: 1+2+3+4

its partial sums are:

$s_0 = 0$

$s_1 = 1$

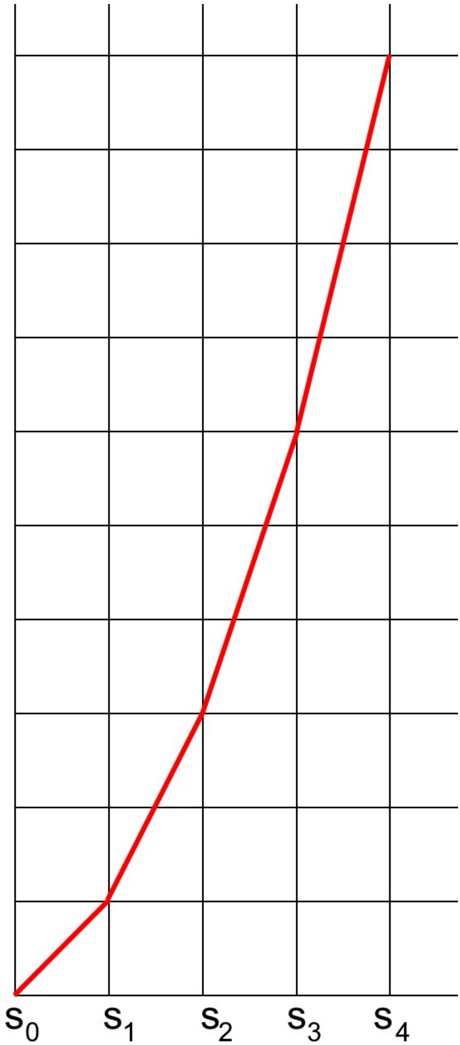
$s_2 = 1+2 = 3$

$s_3 = 1+2+3 = 6$

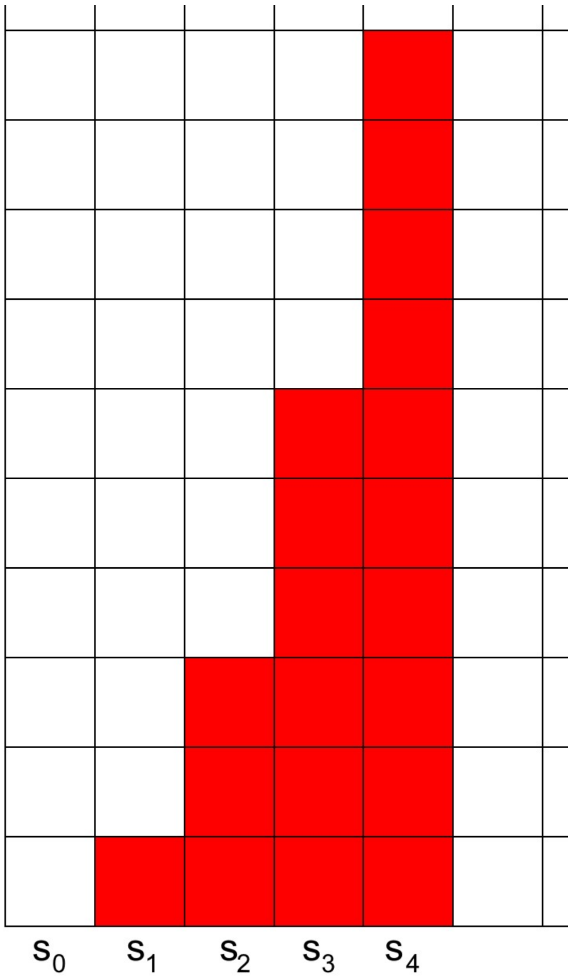
$s_4 = 1+2+3+4 = 10 = s$

The last partial sum coincides with the total sum s.

(Generally, s_0 is not considered as a null sum, but as the sum of only the first term. In this book we will use the convention of s_0: null sum, due to its incredible usefulness).



Line graph of the partial sums of the series 1+2+3+4 (the polygonal chain starts from the origin, which means from the height 0: initial partial sum = 0).



Histogram of the partial sums of the series 1+2+3+4

In the histogram graph of the partial sums the first column must be always empty as it represents the first partial sum which is the null sum (Initial partial sum = 0).

Later we will see how the histogram graph is better than the line graph.

Infinite series

An infinite series is a series with an infinite quantity of terms.

For example, the series which sums every natural number, except 0, is an infinite series:

example: 1+2+3+4+...

1+3+7+9+...

1-1+1-1+...

1+2+2+2+...

1c) Types of infinite series and their characteristics

Convergent series

An infinite series can have a result: the series converges.

example: 1/2 + 1/4 + 1/8 + 1/16 + ... = 1

It's demonstrated that the sum of all the values above (infinite quantity) gives 1. (Later the demonstration).

Alternating series

An alternating series is a series where the signs of the terms alternate progressively between positive and negative (or vice versa).

examples:

1-1+1-1+...

-1+1-1+1-...

1-2+3-4+...

1-4+7-12+3-2+...

1-2+1-2+1-2+...

(For extension we will indicate as alternating series also the series with some 0 terms followed by an alternating series:

example: 0+0+1-1+1-1+...)

An alternating series can converge.

Alternating indeterminate series

Some series do not have a finite result:

example: 1-1+1-1+1-1+... is indeterminate.

The result is indeterminate because the partial sums oscillates between 1 and 0 (we cannot determine which is the final result).

Not all the alternating series are indeterminate, for example the series 1-1/2+1/3-1/4+1/5-... it’s demonstrated that this series converges to ln2.

Divergent series

A divergent series is a series where the sequence of the partial sums diverges. Simply put, “its sum value is infinity”.

example: 1+1+1+1+...

1+4+2+3-8+...

1+2+3+4+... diverge

(1+2+3+4+... = +∞, this way of writing is not rigorous: we have to use instead the limit of the partial sums).

Divergent series with constant terms

Divergent series where all terms are equal

example.: +1+1+1+1+...

+2+2+2+2+...

-2-2-2-2-...

+x+x+x+x+...

Divergent series with non-constant terms, not patterned

Divergent series where the terms are not all equal and they do not follow a pattern.

es.: 1+2+3+4+5+...

1+0+5+60+2+...

Often the methods for the divergent series with non-constant terms can be applied to divergent series with constant terms, because the latter can be considered as a case of the former (like a square can be considered a rectangle with all side of the same length)

Patterned divergent series

A patterned divergent series is a divergent series where the terms follow a patterned, which means the same sequence of terms repeats itself periodically.

Example: 1+2+1+2+1+2+... (pattern: 1, 2)

4+2+1+4+2+1+... (pattern: 4, 2, 1)

1-2+1-2+1-2+... (pattern: 1, -2)

A divergent series with constant terms is a patterned divergent series too: its pattern it's made by only one term.

example: 2+2+2+2+... (pattern: 2)

Thus, the properties of patterned series are valid for the constant-terms divergent series too; but not vice versa.

Non-constant terms series are not patterned series.

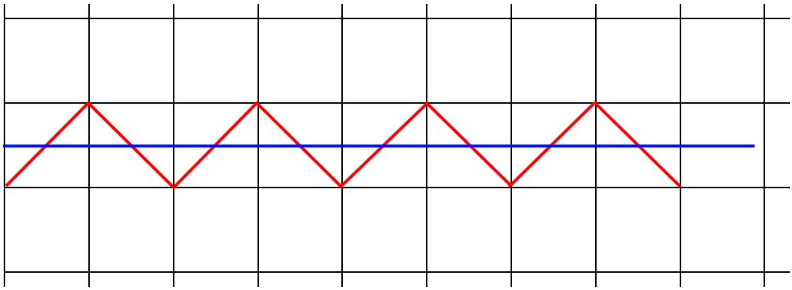
Stable series

When a straight line which passes through a line graph or crosses the steps of the histogram graph of a series is parallel to the x axis, then the series represented by the graph is stable.

The method to find the line which crosses the steps of the histogram of a series is described in the 11th chapter.

We will indicate as “stability line” this line, but sometimes we will extend its meaning to lines which are not parallel to the x axis.

Therefore, the term “stable” in this book is used in a similar meaning (in this case: larger) than the math official meaning.

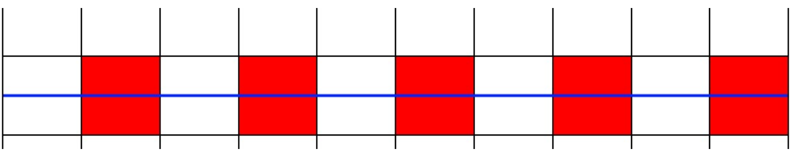


Line graph of di 1-1+1-1+...

The straight line which passes through the graph (blue line) is parallel to the x axis.

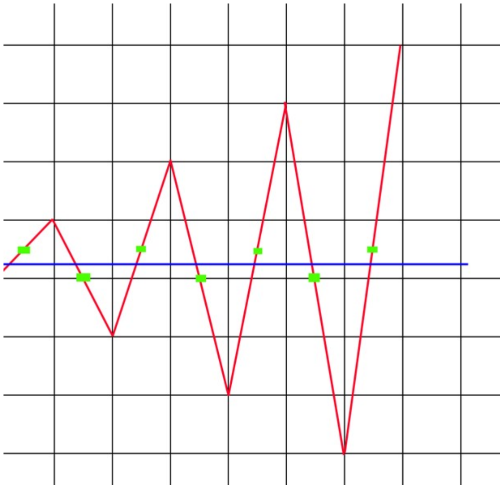
The series 1-1+1-1+... is stable.

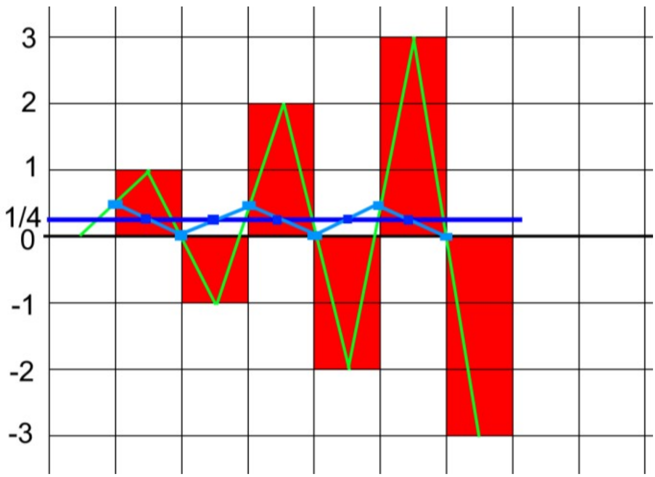
The stability line appears also in the relative histogram graph:



The method to find the blue straight line is described in the 11th chapter.

Line and histogram graphs of the series 1-2+3-4+...





The stability lines in blue colour are parallel to the x axis.

A rigorous system to verify the stability will be described in the 11th chapter.

A stable series can be gathered with the parenthesis in this way:

1-2+3-4+... → 1-(2-3+4-5+...)

More info in the add problem and in the 11th chapter (model).

Geometric series

A geometric series is a series where the ratio of two consecutive terms is constant:

$x_1+x_2+x_3+x_4+...$ ratio = $x_{j+1}/x_j=k$ constant

Example of geometric series:

2+4+8+16+... k=2

1/2+1/4+1/8+1/16+... k=1/2

3+9+27+81+... k=3

Exception terms

Sometimes in a series of a type, might be that 1 or more terms do not follow the main characteristic of the series:

example: 1+1+0+1+1+1+...

This is not a constant terms divergent series, but it's a non-constant terms divergent series.

example: 0+0+1-1+1-1+...

This is still a alternating stable series, but it’s better to precise the presence of 2 exceptional initial zeros.

How to indicate a series and its result

We will indicate often a series with a letter, for example:

a: 1/2+1/4+1/8+1/16+...

in this case a indicates the series 1/2+1/4+1/8+1/16+...

and with a letter its result

example: 1/2+1/4+1/8+1/16+...=s

s=1

s indicates the result of the series (or in general the associated value).

Sometimes, for simplicity, we will use the same letter to indicate the series and its result (or we will use the lowercase and the uppercase letter, or vice versa)

example: A: 1/2+1/4+1/8+...=a

or

a: 1/2+1/4+1/8+1/16+...=A.

In an infinite series it’s not important the number of written terms before the dots, this is important just for the clarity and the understanding for the reader.

Example:

1+2+3+4+5+... is the same thing as 1+2+3+...

Obviously writing 1+... would be very ambiguous because it can mean 1+2+3+4+... or 1+1+1+1+... etc.

1d) Sum of series

Different infinite series can be summed.

They can be summed horizontally or vertically:

Horizontal:

Example:

1+2+3+4+...+1+1+1+1+...

There are many ways of summing more series horizontally and this method is very easy to misunderstand.

Vertically:

Example:

a: 1+2+3+4+... +

b: 1+1+1+1+... =

---

c: 2+3+4+5+...

where c = a+b

The vertical method (in columns) is safe, while the horizontal method can lead to errors or different interpretations and so it’s generally to avoid, or it’s better to precise also the vertical method.

More info in the 5th chapter: “the add problem”.

Empty spaces

If in a sum of more series, a series has empty spaces in the place of the initial terms, these empty places mean the presence of terms of value 0.

a:	1-1+1-1+...	+
b:	2+2+2+...	=
c:	1+1+3+1+...	(c=a+b)

means:

a:	1-1+1-1+...	+
b:	0+2+2+2+...	=
c:	1+1+3+1+...	(c=a+b)

Leaving empty spaces in the place of zeros is a choice to avoid because it’s not clear and can cause oversights (especially in the situation of the add problem, see 5th chapter. In fact, approximately: the 0 in infinite series generally is not a neutral element)

1e) Heuristically associated values

With heuristic methods we can often get finite results for the indeterminate and divergent infinite series too.

The following is an example, we will se the explanation later:

1+2+3+4+5+...=-1/12

The infinity value generated from the sum 1+2+3+4+... can be associated to the finite value -1/12, or even better: the series 1+2+3+4+... can be associated to the value -1/12.

In the next chapter we will see more info about these series.

In the divergent series heuristically treated like 1+2+3+4+...=-1/12 the = is mathematically non right.

In this case the =, from a certain point of view, is used with the meaning of association, and not of equivalence.

In the end there will be a dedicated chapter to some ideas about the model for the description of these results and of the meaning of the = (chapter 11d).

Usually heuristic methods can also be applied to convergent series and the results are right (equal to results obtained by standard methods).

1f) Parenthesis

This topic is delicate. We will use parenthesis to indicate the order of grouping and resolution, for example:

(1+2+3+4+...)+2 → -1/12 + 2

(Where -1/12 is the associated value to 1+2+3+4+...)

1+(1+1+1+1+...) → 1 -1/2

(where -1/2 is the associated value to the series 1+1+1+1+...)

For more info see the chapter 11b (Associative property and gathering)

1g) Famous series

Following, some famous series with their relative associated value.

For some series we will see also the formula of the partial sums, important for further calculation (see 10^{th} chapter: associated functions).

These have been already studied a lot by different intellectuals.

We will study these series almost always with heuristic methods.

These heuristic results and methods for these famous series will be considered, in this book as true and as starting points.

1/2+1/4+1/8+1/16+...=1

There are heuristic and non-heuristic methods to solve this series.

Graphic intuition

The graph of the partial sums gets closer progressively to the height y=1: in every step a term is added (1/2, 1/4, 1/8, etc.).

Non heuristic

The partial sums of the series are:

s_0 = 0

s_1 = 1/2

s_2 = 1/2+1/4 = 3/4

s_3 = 1/2+1/4+1/8 = 7/8

s_4 = 1/2+1/4+1/8+1/16 = 15/16

...

The partial sums follow the following formula:

$$y = \frac{2^t - 1}{2^t} = 1 - \frac{1}{2^t}$$

Where t is the index of the partial sum.

With t tending to infinity, the value of the partial sum tends to 1 (we use the limit).

Heuristic

1/2 + 1/4 + 1/8 + 1/16 +... = s

1/2 (1+ 1/2 + 1/4 + ...) = s

1/2 (1 + s) = s

s=1

$x+x^2+x^3+x^4+... = x/(1-x)$

In maths this series is presented with the condition: $0 \le x < 1$

The previous example can be described with this formula too. In fact: we can see that $x=1/2$, $x^2=1/4$, $x^3=1/8$, etc..

Hence the formula works:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots=\frac{\frac{1}{2}}{1-\frac{1}{2}}=1$$

Partial sums

The formula for the partial sums is:

$$y=\frac{a-a^{t+1}}{1-a}$$

where t is the index of the partial sum.

With a=1/2 and t tending to +infinity, y is 1

Heuristic

$x+x^2+x^3+x^4+...$ s

$x(1+x+x^2+x^3+...)=s$

x(1+s)=s

x+xs=s

x=s-xs

s=x/(1-x)

We can heuristically extend this formula (and similar ones) for all numbers

for example:

2+4+8+16+...=-2

2+4+8+16+...=s

2(1+2+4+8+...)=s

2(1+s)=s

2+2s=s

s=-2

we can notice that: x/(1-x) with x=2 becomes:

2/(1-2) = -2

$1+x+x^2+x^3+... = 1/(1-x)$

Partial sums

The formula for the partial sums is the formula of $x+x^2+x^3+...$ plus 1:

$$y = \frac{a - a^{t+1}}{1-a} + 1 = \frac{1 - a^{t+1}}{1-a}$$

where t is the index of the partial sum and a is the term of the geometric series

Heuristic

$1+x+x^2+x^3+...=s$

$1+x(1+x+x^2+x^3+...)=s$

$1+xs=s$

$1=s-xs$

$1=s(1-x)$

$s=1/(1-x)$

(This series is 1 + the series $x+x^2+x^3+...$ hence the sum of the associated values is:
$1+x/(1-x) = 1/(1-x)$)

Example:

$1+2+4+8+16+...=-1$

Falls in the previous case:

$1+2+4+8+16+...=s$

$1+2(1+2+4+8+...)=s$

$1+2s=s$

$s=-1$

we can notice that with $x=2$ $s=1/(1-2)=-1$

$x-x^2+x^3-x^4+...=x/(1+x)$

Heuristic

$x-x^2+x^3-x^4+...=s$

$x(1-x+x^2-x^3+...)=s$

$x(1-s)=s$

$x-xs=s$

$x=s+xs$

$x=s(1+x)$

$s=x/(1+x)$

$2-4+8-16+...=2/3$

$2-4+8-16+...=s$

$2(1-2+4-8+...)=s$

$2(1-s)=s$

$2-2s=s$

$2=3s$

$s=2/3$

we can notice that x/(1+x) with x=2 is:

$2/(1+2)=2/3$

$1-x+x^2-x^3+...=1/(1+x)$

$1-x+x^2-x^3+...=s$

$1-x(1-x+x^2-...)=s$

$1-xs=s$

$s+xs=1$

$s=1/(1+x)$

(This series can be see as 1 minus the series $x-x^2+x^3+...=x/(1+x)$ so:

$1 - x/(1+x) = 1/(1+x)$)

Example:

$1-2+4-8+...=1/3$

$1-2+4-8+...=s$

$1-2(1-2+4-8+...)=s$

$1-2s=s$

s=1/3

so 1/(1+x) with x=2 becomes 1/(1+2)=1/3

1-1+1-1+...=1/2

This series in maths is alternating and does not converge.

It’s called Grandi’s Series and it’s one of the most famous series in the heuristic landscape.

Studying heuristically the partial sums

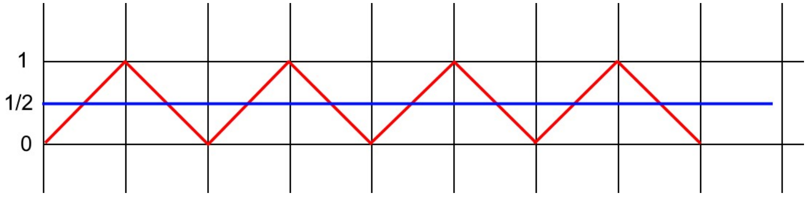
The partial sums of this series are:

1, 0, 1, 0, ...

Thus, the final result is an average of the two possible partial sums:

s = (1+0)/2 = 1/2

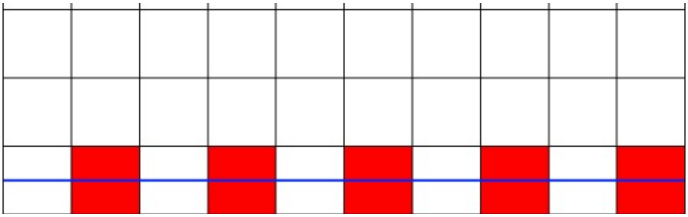
Graphic intuition



The line graph shows the partial sums: the initial point is the origin, since before the first +1 the partial sum is 0; then the +1 makes the partial sum 1, and then the -1 makes it 0, and so on.

Actually, a line graph is not very appropriate, since the transition from a partial sum to another partial sum is gradual (but actually a sum is instantaneous).

To avoid this problem, we need to use a histogram (although the line graph is often quicker and easier to use and analyse).



The blue straight line which cross the graph (y=1/2) is parallel to the x axis, hence the series is stable.

The word “stable” in this book is not used with the same meaning of the maths one, although it’s similar.

Later we will try to define it.

(The methods to analyse graphs and to verify the stability of a series and to extract the value of a series from its graph, see chapter 11)

The first column is empty, since the initial state (before the first +1) is 0 (the histogram shows the partial sums).

Numerical

1-1+1-1+...=s

1-(1-1+1-...)=s

$1-s=s$

$1=2s$

$s=1/2$

Relation with $x-x^2+x^3-x^4+...$

The series 1-1+1-1+... can be described also with the formula $x-x^2+x^3-x^4+...=x/(1+x)$: with x=1 it becomes $1/(1+1)=1/2$

Physical intuition – mental experiment

Let’s imagine a lamp which turns on and off fast: the evolution is described by the series 1-1+1-1+...:

on every step (every time interval):

+1: the lamp turns on (luminosity increases of 100%) thus: partial sum = 1: light on

-1: the lamp turns off (luminosity decreases of 100%) thus: partial sum = 0: light off

The lamp turns on and off in this way forever.

Let’s suppose that there is a video recording of the lamp.

The video has an infinite length, but let’s suppose that is possible to view it and manage it.

A person is interested in discovering what is the “final” state of the lamp.

The person decides to accelerate the video too see what is the final state.

The person progressively accelerates more the video, and therefore see the lamp is flickering: the states light on and light off starts to become indistinguishable.

In the end the video is accelerated by the factor infinite (video reproduction velocity x ∞).

The frames are very fast and indistinguishable and they mix themselves, the luminosity of the lamp appears a middle way between light on (luminosity=1) and light off (luminosity=0): luminosity = 1/2

If a light blink fast, we perceive an intermediate luminosity.

The person come to 2 conclusions:

1) The lamp has a final state of an intermediate luminosity (luminosity = 0.5)

2) The lamp had, from a non-temporal point of view, a luminosity of 0.5.

This story is similar to the famous puzzle the “Thomson’s lamp”.

1-2+3-4+5-6+...=1/4

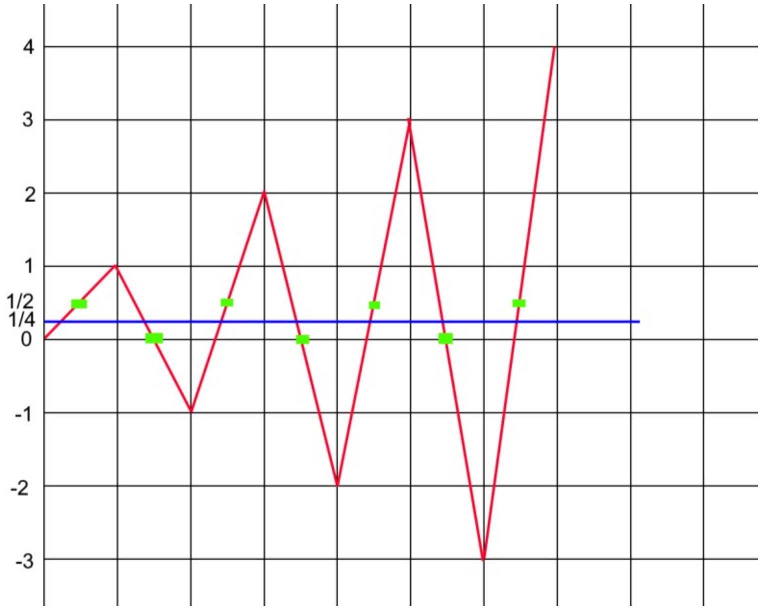
This series is in maths an alternating and not convergent series and it’s one of the most famous series in the heuristic landscape.

Partial sums

The partial sums are:

$s_0 = 0$ $s_1=1$ $s_2=-1$ $s_3=2$ $s_4=-2$ ...

Graphic intuition



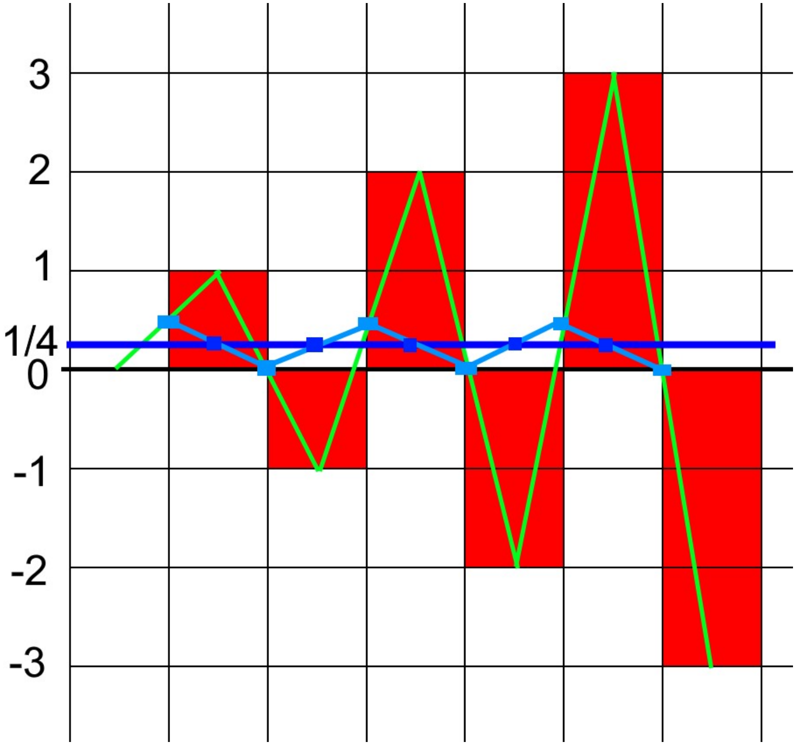
The blue straight-line y=1/4 “summarize” the graph. It intersects the y axis in the ordinate 1/4.

The green markers show the middle points of every red segment, the blue line is the centre line of the zone delimitated by the green markers.

We can notice that the series is stable, as a matter of fact, the line y=1/4 is parallel to the x axis.

(The term stable is used in this book in a different meaning than the standard math meaning)

Histogram graph:



In the histogram:

- The green polygonal chain has as vertexes the midpoints of the horizontal sides of the columns of the graph.
- The light blue makers indicate the midpoints of the segments of the green polygonal chain.
- The light blue polygonal chain connects the light blue markers
- The blue markers indicate the midpoints of the segments of the light blue polygonal chain

The blue straight line is parallel to the x axis, thus the series represented by the graph is stable (in the meaning used in this book).

In the 11th chapter there will be a partial explanation of the method to extract the value of a series from its graph (like in the example just saw).

Let’s continue the explanation for the series:
1-2+3-4+...=1/4

First numerical explanation

1-2+3-4+5-6+... = s   +

0+1-2+3-4+5-... = s   (due to stability) =

1-1+1-1+... = 2s

But 1-1+1-1+...=1/2 (see previous pages)

Thus 1/2=2s , hence:

1-2+3-4+...=1/4

Second numerical explanation

1-2+3-4+...=s

1-(2-3+4-5+...)=s

1-(1-2+3-4+...   +

+1-1+1-1+...)=s

1-(s+1/2)=s

1-s-1/2=s

1/2=2s

s=1/4

1+1+1+1+...=-1/2

This series in maths is divergent, and it’s one of the most famous in the heuristic landscape.

Partial sums

Partial sums are:

$s_0 = 0$ $s_1 = 1$ $s_2 = 2$ $s_3 = 3$ $s_4 = 4$ …

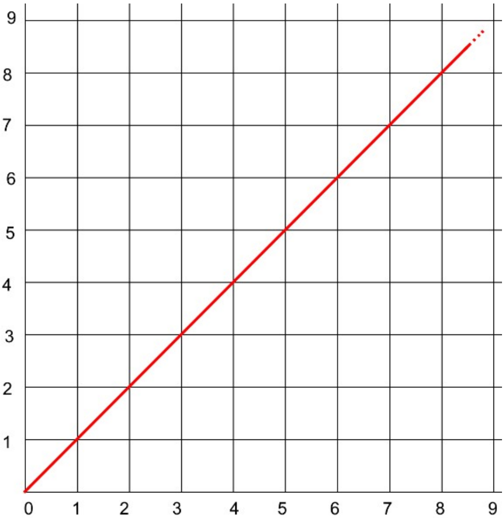
And they follow the formula:

y=x

where x is the index of the partial sum.

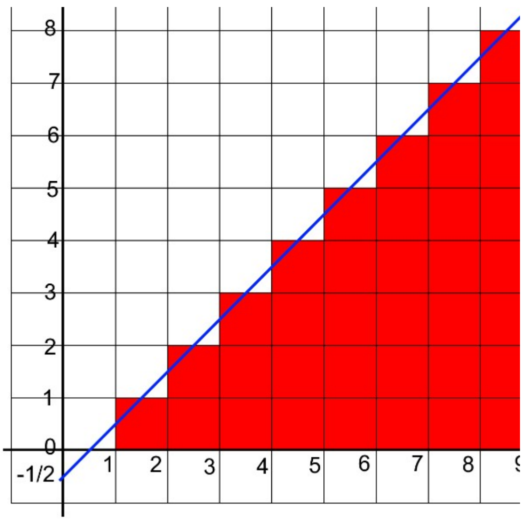
Graphic intuition

In this case line graphs are not efficient, as we can see:



In this case the line graph of the partial sums of 1+1+1+... intersects the y axis in 0.

Instead, the histogram graph brings to the value -1/2:



The straight line (which is the "summarizing line" which is the associated function (see 9^{th} and 10^{th} chapters)) cuts the steps and passes through the middle points of the horizontal sides of each column, and intersects the y axis in the -1/2 ordinate which corresponds to the result we will find with a numerical procedure too.

Numerical motivation

The "demonstration" is similar to the one of Ramanujan (which we will see later):

a: +1+1+1+1+... = s +

b: +0-2+0-2+... = -2s =

---

c: +1-1+1-1+... = -s

But 1-1+1-1+... = 1/2 hence -s=1/2 therefore s=-1/2 so

1+1+1+1+...=-1/2

Relation with the series $x+x^2+x^3+...$

The series 1+1+1+1+... can be seen as the series $x+x^2+x^3+...$ with x = 1, but the formula for the associated value does not work: with x=1 the condition of the domain of the function x/(1-x) are not satisfied (x/(1-1))

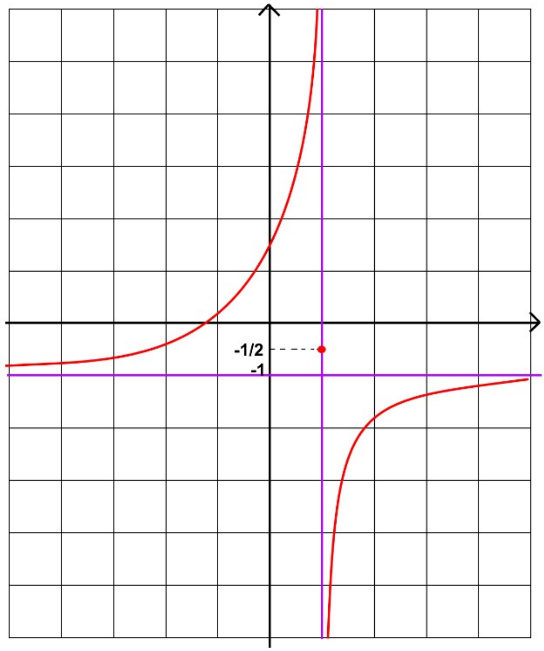
So the series $x+x^2+x^3+...=x/(1-x)$ with x=1 is not defined.

If we calculate the limit for 1^+ and 1^- we can notice that:

$$\lim_{x \to 1^+} \frac{x}{1-x} = -\infty$$

$$\lim_{x \to 1^-} \frac{x}{1-x} = +\infty$$

For x=1 we can “force” the presence of the value -1/2: following a qualitative graph of x/(1-x) with the value -1/2 in x=1:



In violet the vertical asymptote (x=1) and horizontal one (y=-1).

<u>The situation seems to me asymmetric.</u>

<u>Further explanations of the reason why the formula $x+x^2+x^3+...=x/(1-x)$ has a break of its behaviour in x=1 might bring a better comprehension of the infinite series.</u>

1+2+3+4+5+... = -1/12

Partial sums

The partial sums are:

$s_0 = 0$
$s_1 = 1$
$s_2 = 1 + 2 = 3$
$s_3 = 1 + 2 + 3 = 6$
$s_4 = 1 + 2 + 3 + 4 = 10$
…

The sum follows the function:

$$y = \frac{x(x+1)}{2} = \frac{x^2}{2} + \frac{x}{2}$$

Heuristic associated value

There are different possible procedures, following the most famous one. The procedure is attributed mainly to the mathematician Ramanujan.

1+2+3+4+5+...=s

s-4s=-3s

(1+2+3+4+...)-4(1+2+3+4+...)=-3s

1+2+3+4+...-4-8-12-16-...=-3s

+1+2+3+4+5+6+... = s

-4-8-12-16-... = -4s

dilating the second series of 2:

0-4+0-8+0-12+0-16+...=-4s

Summing the two series in columns:

+1+2+3+4+5+6+... = s   +

+0-4+0-8+0-12+...=-4s   =

1-2+3-4+5-6+...=-3s

But 1-2+3-4+...=1/4 as we already saw before, hence:

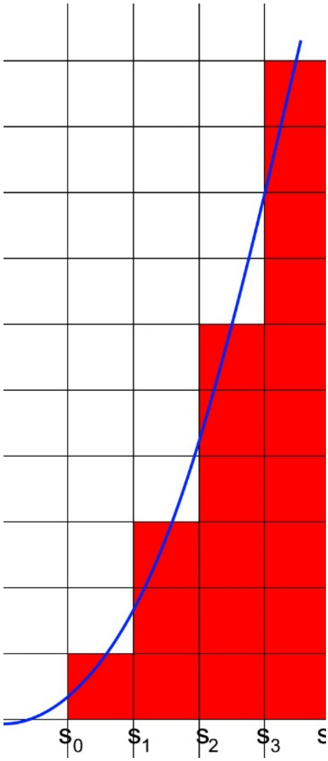
1/4=-3s

s=-1/12

1+2+3+4+5+...=-1/12

Graphic intuition

In the following graph, the curve passes through all steps (see chapters 10 and 9 for more info about the analysis and value extraction in graphs) and intersects the y axis in -1/12.



The graph is not very precise. It’s been done with a simple drawing software, and it’s only qualitative.

At the moment it’s not clear to me the method to use to generate the blue curve in all similar cases (the method used for 1+1+1+... and for 1-2+3-4+... doesn’t work), however in this particular case I found that the curve follows the function:
$y=x^2/2 - 1/12$

You can find a better graph on Wikipedia:
(https://en.wikipedia.org/wiki/1_%2B_2_%2B_3_%2B_4_%2B_%E2%8B%AF)

1+4+9+16+...=0

Partial sums

$$s_0 = 0$$
$$s_1 = 1$$
$$s_2 = 1 + 4 = 5$$
$$s_3 = 1 + 4 + 9 = 14$$
$$s_4 = 1 + 4 + 9 + 16 = 30$$
$$\dots$$

The partial sums follow the function:

$$y = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

The calculation is similar to the one for the series 1+2+3+4+... and 1+1+1+1+...:

1+4+9+16+...=s

s-8s=-7s

1+4+9+16+25+... = s

-8(1+4+9+16+25+...)= -8s

1+4+9+16+25+36+... = s

-8-32-72-... = -8s

Dilating the second series by 2 and summing the two series in vertical:

+1+4+9+16+25+36+... = s

+0-8+0-32+0 -72-... = -8s

+1-4+9-16+25-36+... = -7s

This last series (1-4+9-16+...) gives the value 0 (see chapter 2b).

Therefore -7s=0 → s=0 → 1+4+9+16+...=0

The series 1+4+9+16+... is the Riemann Zeta of 2, like the series 1+2+3+4+... is the Riemann Zeta of 1, and the series 1+1+1+1+... is the Riemann Zeta of 0. See chapter 8 for further explanations about Riemann Zeta.

Value extraction techniques

The partial grouping factorization can be used in some cases as for example in the geometric series:

e.g.: 1+2+4+8+16+... = s

1+2(1+2+4+...) = s

But not in those non-geometric and non-stable (further information in the add problem: chapter 5)

e.g.: 1+2+2+2+2+... = s

1+2(1+1+1+...) NO!

The comparison to other series is possible, but only in columns (vertical approach) or if in line, based on the vertical one:

e.g.: 1-1+1-1+... = 1/2 +

1-2+3-4+... = 1/4 =

2-3+4-5+... = 3/4

2-3+4-5+... = 1-1+1-1+... +1-2+3-4+...

Other series

Based on the series described above, we can, by applying simple maths rules, find other series and their associated very important values:

a-a+a-a+...=a/2

for example, 3-3+3-3+...=3/2

a+a+a+a+... = -a/2

for example, 3+3+3+...=-3/2

n(1+2+3+4+...) = -n/12

2+4+6+8+10+...=-1/6

and generally if $x_1+x_2+x_3+...=r$ then $n(x_1+x_2+x_3+...)=nr$

1h) Associated functions and generatrix series

Function of terms and of partial sums

For a series it can be found a function of the terms and a function of the partial sums.

The function of the terms indicates the behaviour of the terms, the function of the partial sums indicates the terms of the partial sums.

Example:

For the series 1+2+3+4+5+...

the function of the terms is f(x)=x (with x natural number except 0), the terms follow indeed this function.

The function of the partial sums is g(x)=x(x+1)/2 (with x natural number except 0), the partial sums follow indeed this function.

Generatrix series

The series which generates as partial sums a sequence of numbers, is called the generatrix series of that sequence.

Example:

0, 1, 0, 1, 0, 1, 0, ...

(The first term: 0, is the null sum)

has as generatrix series:

1-1+1-1+1-1+...

Or:

0, 1, -1, 2, -2, 3, -3, ...

has as generatrix series:

1-2+3-4+5-6+...

Other associated functions

We can associate a function to a series where the function will intersect y axis in the ordinate equal to the value extracted via heuristic methods from the series.

We will see these functions often showed in the histogram graphs, as a general behaviour line, “summarizing line”.

The method to find this function is not clear to me, but we will see a few cases more in depth in the 10th chapter (Graphs, stability detection and value extraction) and in the 9th chapter (Associated functions).

Connection between values and functions

In this book we will start focusing on the process:

series → values

then we will see a little of the process:

series → functions → values

In my opinion, if a solidity in the connection between series and values is verified, thus the associated values gain validity or importance, and the heuristic maths models can become a starting point for an update of maths.

2) Sliding

2a) Sliding

The sliding is the insertion of 0s in front of a series.

Following a series, its slid by 1, and its slid by 2.

a: 1+2+3+4+5+...

a’:   0+1+2+3+4+...

a’’: 0+0+1+2+3+...

We indicate sometimes with 1 apostrophe (a’) the series slid by 1 term, with 2 apostrophes (a’’) the series slid by 2 terms and so on.

If we indicate with d the number of 0s in front of a series, d is the sliding number, and the series is slid by d.

e.g.: a’’: 0+0+1+2+3+...   d=2

Let’s see now the values of the slid series compared to the original ones:

ALTERNATING STABLE

Due to stability the result does not change.

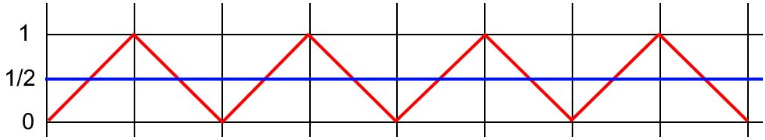
A: 1-1+1-1+1-1+... = 1/2

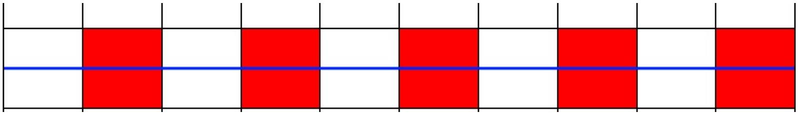
A’:   0+1-1+1-1-1+... = 1/2

A’’: 0+0+1-1+1-1+... = 1/2

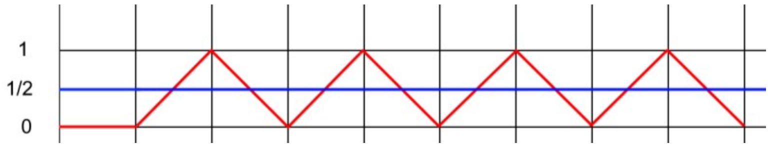
The value of A’ is equal to that of A because A is stable.

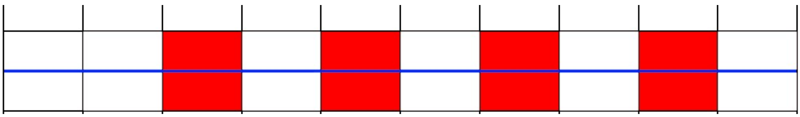
Line graph and histogram graph of
1-1+1-1+... :



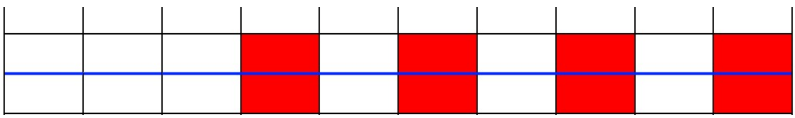


Line graph and histogram graph of
0+1-1+1-1+...:





Line graph and histogram graph of
0+0+1-1+1-1+...

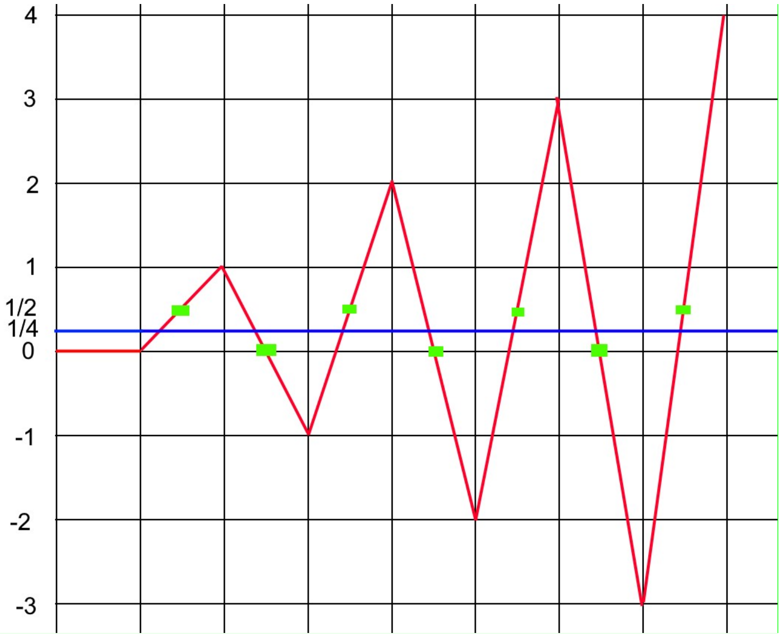


The line which cuts the graph intersects in all the three cases the y axis in the ordinate 1/2 (1-1+1-1+...=1/2, 0+1-1+1-1+...=1/2, 0+0+1-1+1-1+...=1/2) and it's parallel to the x axis. The series are thus stable.

The crossing blue line can be obtained with the procedure described in the 10th chapter.

A: 1-2+3-4+... = 1/4

A’: 0+1-2+3-4+... = 1/4



A’’: 0+0+1-2+3-4+5-6+... = 1/4

In this second example I inserted only the line graph for simplicity.

For further explanation about the verification of the stability of a series from the partial sums or from the graph and about the extraction of the associated value, see 10th chapter.

Let A be an alternating stable series, A slid by n ($A^{(n)}$) has the same value of A.

DIVERGENT CONSTANT TERMS

The problem falls in "the add problem" explained better later (see 5th chapter)

In general:

If A is a divergent series with constant terms, A slid by n has NOT the same value of A.

Example:

1+1+1+1+... = -1/2

0+1+1+1+... = -3/2

0+0+1+1+... = -5/2

The value can be calculated via vertical approach (see 5th chapter):

example to get 0+1+1+1+...=-3/2:

+1+1+1+1+...=-1/2 +

-1+0+0+0+...=-1 =

0+1+1+1+...= -1/2-1 = -3/2

Notice how the 0 in infinite series "is not the null element of the addition".

Actually, one can object that the sum in this series, which are heuristically treated, are not real proper sums; or that the = has not a real meaning of equivalence.

Generalisation of the sliding for divergent constant terms series

In the light of the resolution of the add problem, the sliding for divergent series with constant terms can be generalized in this way:

For s: 1+1+1+...=-1/2 slid by n terms:

0+...+0+1+1+1+... (with n initial 0s):

$s^{(n)}$: 0+...+0+1+1+1+... = -1/2-n

And in general, for s: a+a+a+...=-a/2 slid by di n terms:

0+...+0+a+a+a+... (with n initial 0s):

$s^{(n)}$: 0+...+0+a+a+a+... =-a/2-na

DIVERGENT NON-CONSTANT TERMS, NON-PATTERNED

This case falls in “the add problem” too:

In general:

If A is a divergent series with non-constant terms, A slid by n will not have, usually, the same value.

Example:

1+2+3+4+5+...=-1/12

0+1+2+3+4+...= 5/12

0+0+1+2+3+4+...= 23/12

Further explanation in the 5th chapter “the add problem” and 9th chapter “associated functions”

Geometric series do not change value if slid, although they are divergent with non-constant terms

0+2+4+8+16+... = -2

2(0+2+4+8+16+...)=-2*2

0+4+8+16+...=-4

Summing in columns (further explanation in the 5th chapter):

0+4+8+16+... = -4 +

2+0+0+0 +... = 2 =

2+4+8+16+... = -2

0+0+2+4+8+16+...=-2

4 (0+0+2+4+8+16+...) = -2*4

0+0+8+16+32+... = -8

Summing in columns:

0+0+8+16+32+... = -8

2+4+0+0 +0 +... = 6

2+4+8+16+32+... = -2

$0+x+x^2+x^3+... = x/(1-x)$

$x(0+x+x^2+x^3+...) = x^2/(1-x)$

$0+x^2+x^3+x^4+... = x^2/(1-x)$

Summing in columns:

$0+x^2+x^3+x^4+... = x^2/(1-x)$

$x+0+0+0+0+... = 0$

$x+x^2+x^3+x^4+... = x^2/(1-x)+x$

$x+x^2+x^3+x^4+... = x/(1-x)$

$x+x^2+x^3+x^4+...=s \rightarrow 0+...+0+x+x^2+x^3+x^4+...=s$

The reason why geometric series, even if divergent and with non-constant terms and non-patterned, do not change value if slid is still unclear to me and difficult to understand.

An attempt to explain it will be in the 9th chapter.

Connection with the dilation

Connecting to the dilation, we can notice how 0+1+0+1+0+1+... is the 1-slid of 1+0+1+0+1+0+...

More info in the 6th chapter.

2b) Sliding sum

The sliding sum is a sum of a series with itself slid by n positions.

Hence, it’s a special case of the sum of two series, where the second is the same as the first but with one or more zeros in front of the first term.

The two series must be summed vertically (which means in columns).

It has a great utility for the value extraction in the stable series (since their stability ensures the series keeps the same value even if slid).

Let A: 1-1+1-1+... = 1/2

and A’: 0+1-1+1-1+...

let’s find A+A’ (let’s use the vertical approach!)

A:	1-1+1-1+...	= 1/2	+
A’:	0+1-1+1-...	= 1/2	=
A+A’:	1+0+0+0+...	= 2*1/2 = 1	

Coherent!

By summing a series with its slid we get a coherent result.

Another example:

Let A: 1-1+1-1+... = 1/2

and A'': 0+0+1-1+1-1+...=1/2

A:	1-1+1-1+1-...	= 1/2	+
A'':	0+0+1-1+1-...	= 1/2	=
A+A'':	1-1+2-2+2-2+...	= 1	

Let's verify with another methods if

1-1+2-2+2-2+...=1

The series 2-2+2-2+... is stable, hence it can be grouped.

1-1+(2-2+2-2+...)=s

2-2+2-2+...=1 hence

1-1+1=s

1=s Coherent.

1-2+3-4+5-6+...=1/4

Since 1-2+3-4+... is stable we can find its value by using a few times sliding sums, that is to say:

1-2+3-4+5-6+...=s	+
0+1-2+3-4+5-...=s (because it's stable)	=
1-1+1-1+1-1+...=2s	

And 1-1+1-1+... falls into the case described above:
1/2=2s

s=1/4

1-4+9-16+25-36+...=0

The series is stable and in the line graph the stability line is parallel to x axis.

In the next page the graph of 1-4+9-16+...

Line graph of 1-4+9-16+...

The line which cuts the graph is y=0 and is parallel to x axis, therefore the series is stable.

We can thus apply the sliding without changing the value of the series.

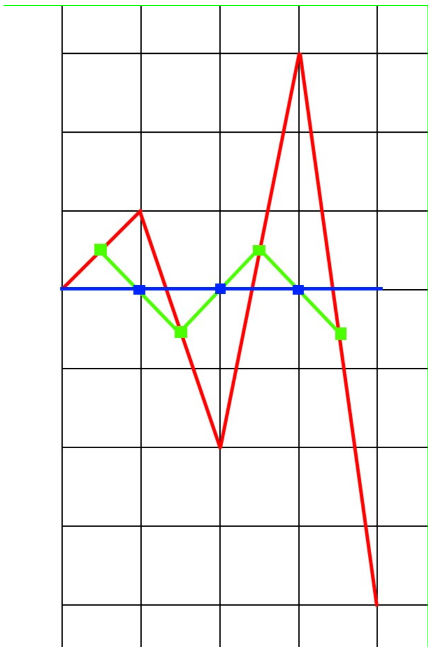
1-4+9-16+...=s   +

0+1-4+9 -...=s   =

1-3+5-7+...=2s=z

By applying again the sliding sum:

1-3+5-7+...=2s=z



Line graph of 1-3+5-7+... (The line which cuts the graph is y=0 and is parallel to x axis, hence this series is also stable)

1-3+5-7+...= 2s = z   +

0+1-3+5-...= 2s = z =

1-2+2-2+...= 4s = 2z

1-2+2-2+2-...= 4s = 2z (1-2+2-2+... è stabile) +

0+1-2+2-2+...= 4s = 2z =

1-1+0+0+0+...= 8s = 4z

1-1=0 hence 8s = 0 thus s=0 therefore

1-4+9-16+25-36+...=0

and also

1-3+5-7+9-11+...=0

Generally, in order to find the value of a stable series it is only needed to apply the sliding sum, and applying it again to the resulting series, until the result will be finite.

Properties:

stable series + stable series = stable series

Where the sum is made vertically (vertical approach)

A situation where it is wrong to apply the sliding sum.

1-2+1-2+1-2+...=y

0+1-2+1-2+1-...= NOT y

The sliding sum is not easy to apply, since 1-2+1-2+... is not stable hence a sliding will change its value.

2c) Anti-sliding

We call the reverse process of the sliding anti-sliding or sliding by a negative integer factor.

E.g.:

starting from 0+1+1+1+1+...

the anti-slid by 1, or slid by -1, is:

1+1+1+1+...

Connection with the dilation

Connecting to the dilation, we notice that, for example, the series 0+1+0+1+0+1+... is the (-1)-slid, that is anti-slid of 1, of the series 1+0+1+0+...

More info in the 6th chapter.

2d) Sliding general formula

We already saw the sliding by a negative integer number (see previous paragraph).

Moreover, the sliding can be generalized via associated functions for all real numbers (see 9th chapter)

Interesting will be understand which are, if they exist, the numerical representation or in series of the slid series by a non-integer factor.

3) Compression

The compression method is a good method to solve some series.

A compression can be applied by a factor of 1, 2, 3, 4, ...

It involves partial sums of the series, in different ways (later explained), as many as the compression factor; the average of the different resolution is then done: the average is equal to the associated value to the starting series.

The partial resolutions of the series use different grouping and sums of terms.

The compression is visualizable very easily with line graphs (the representation via histogram graphs is less understandable)

3a) 2-compression

The 2-compression is useful in many occasions.

The 2-compression can be also called Up-Down Compression due to its graph (later shown) which shows two lines: one above and one under the main graph.

The 2-compression identifies 2 ways of partial resolution of a series, obtaining 2 new series, probably easier, then we make the average of the 2 series (vertically) or of the two associated values to the two new series.

The average of the two, implicates often the elision of components, which if removed, bring the calculation to a finite quantity of terms.

It's still not clear to me the general criterion to understand if the compression can be applied to a series.

The compression can be applied to series which are:
- stable
- with constant terms
- patterned

Not to the divergent series with non-constant terms and non-patterned though.

I did not verify if it's possible to apply the compression to geometric series.

Following some examples of every case:

ALTERNATING STABLE

The 2-compression can be applied to the alternating stable series to extract the associated value.

1-2+3-4+...=1/4

Here are the 2 ways of grouping in pair (2 terms since it's 2-compression) the series terms:

U: 1+(-2+3)+(-4+5)+(-6+7)+...

D: (1-2)+(3-4)+(5-6)+...

then we sum the terms inside the parenthesis:

U: 1+(1)+(1)+(1)+...

D: (-1)+(-1)+(-1)+...

then we divide in n parts the terms resulting from the pair sums, where n is the compression factor (in the U series, the first term must not be divided since it's not coming from inside parenthesis).

U: 1 + (1/2 + 1/2) + (1/2 + 1/2) + (1/2 + 1/2) +...

D: (-1/2 -1/2) + (-1/2 -1/2) + (-1/2 -1/2) +...

Let's remove the parenthesis and let's do the average of the two series (summing in columns and then dividing by 2):

U: +1 +1/2 +1/2 +1/2 +... +

D: -1/2 -1/2 -1/2 -1/2 -... =

1/2 +0 +0 +0

and dividing by 2 we have: 1/4

1-2+3-4+... = 1/4 Coherent

The result is coherent with the other types of calculation.

Moreover, if the two new series (U and D) are solved with the add method (we will see it later) we will get:

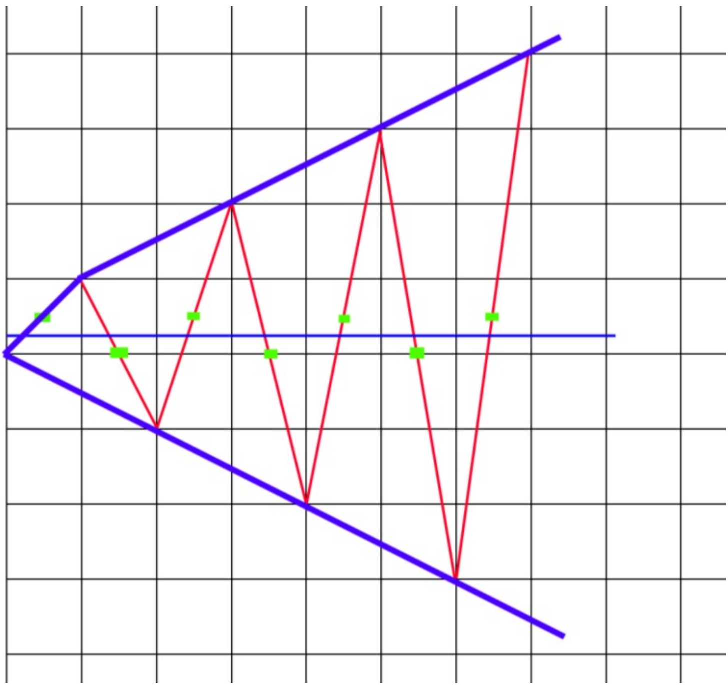
u=1/4 and d=1/4 whose average gives coherently 1/4.

+1	+1/2	+1/2	+1/2	+...	=	1/4	+
-1/2	-1/2	-1/2	-1/2	-...	=	1/4	=
1/2	+0	+0	+0	+...			/2

1/4

Coherent in both members.

Graphic illustration



Visualization of the up and down compressed series.

The upper part of the blue line (the first little square actually coincides with the red line) is the U series (Up series: 1+1/2+1/2+1/2+...), while the part under the red graph is the D series (Down series: -1/2-1/2-1/2-...).

It's very easy to use the line graph to represent the compression process.

In general, for alternating stable series, in the 2-compression process:

$a_1 - a_2 + a_3 - a_4 + ...$

U is the Up compression, D is the Down compression

S = (U+D)/2 = (u+d)/2

where u and d are the values associated to the series U and D

The compression almost simulates the process of upper and lower surrounding of a graph, and this is similar to the overestimation and underestimation typical of the algorithm for the calculation of the integrals.

Another example:

1-1+1-1+... = 1/2

U: 1+(-1+1)+(-1+1)+...

D: (1-1)+(1-1)+...

U: 1+(0)+(0)+(0)+...

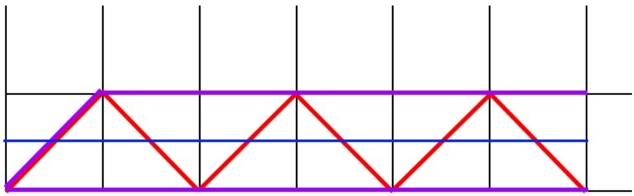
D: (0)+(0)+(0)+...

U: 1+0+0+0+0+... = 1

D: 0+0+0+0+0+... = 0

Average: 1/2 Coherent!

Graphic illustration:



The upper violet line (in the first square coincides with the red graph) is the line which represent the Up series (1+0+0+0+...), while the lower violet line represents the Down series (0+0+0+...)

One more example:

1-3+5-7+9-11+...=0

U: 1+(-3+5)+(-7+9)+(-11+...

D: (1-3)+(5-7)+(9-11)+...

U: 1+(2)+(2)+(2)+...

D: (-2)+(-2)+(-2)+...

U: +1+1+1+1+1+1+1+... = -1/2 +

D: -1-1-1-1-1-1-1-... = 1/2 =

U+D: 0

(U+D)/2 = 0

1-3+5-7+9-11+...=0

DIVERGENT, CONSTANT TERMS

It’s trivial: for 1+1+1+1+... (=-1/2) we notice that:

U: 1+(2)+(2)+...

D: (2)+(2)+(2)+...

U: 1+(1+1)+(1+1)+...

D: (1+1)+(1+1)+(1+1)+...

U: 1+1+1+1+1+... +

D: 1+1+1+1+1+... =

U+D: 2+2+2+2+...

S=(U+D)/2 = 1+1+1+1+...

The resulting series is identical to the starting series, hence applying the compression irrelevant.

DIVERGENT, NON-CONSTANT TERMS, NON PATTERNED

At the moment I think the compression can't be applied to divergent series with non-constant series and which are not patterned, with efficient and useful results.

The only exception might be with the geometric series, but I still did not verify this case.

3b) 3-compression

ALTERNATING STABLE

Example:

1-2+3-4+5-6+7-8+9-...=1/4

A: (1-2+3)+(-4+5-6)+(7-8+9)+...

B: +1+(-2+3-4)+(5-6+7)+(-8+9-...

C: +1-2+(3-4+5)+(-6+7-8)+9

A: (2)+(-5)+(8)+...

B: +1 +(-3)+(6)+...

C: 1 -2 + (4) + (-7) +...

A:	2/3	+2/3	+2/3	-5/3	-5/3	-5/3	+8/3	+...	+
B:	1		-3/3	-3/3	-3/3	+6/3	+6/3	+6/3 +...	+
C:	1		-2	+4/3	+4/3	+4/3	-7/3	-7/3 +...	=

8/3 -7/3 +3/3 -4/3 +5/3 -6/3 +7/3 +...

The series we get is:

8/3 -7/3 +3/3 -4/3 +5/3 -6/3 +7/3 -...=

1/3 (8-7+3-4+5-6+7-...) =

1/3 (7-5+0+0+0+0+0+...

1-2+3-4+5-6+7-...)=

1/3 (2+1/4) = 1/3 (9/4) = 3/4

(3/4)/3 = 1/4

Coherent!

DIVERGENT, CONSTANT TERMS

1+1+1+1+...=1/2

Trivial.

A: (1+1+1)+(1+1+1)+(1+1+1)+...

B: 1+(1+1+1)+(1+1+1)+(1+1+1)+...

C: 1+1+(1+1+1)+(1+1+1)+(1+1+1)+...

A: (3)+(3)+(3)+...

B: 1+(3)+(3)+(3)+...

C: 1+1+(3)+(3)+(3)+...

A: 1+1+1+1+1+...

B: 1+1+1+1+1+...

C: 1+1+1+1+1+...

The average is the starting series, the compression is irrelevant.

DIVERGENT, NON-CONSTANT TERMS, NOT PATTERNED

At the moment I think the compression can’t be applied to divergent series with non-constant series and which are not patterned, with efficient and useful results.

The only exception might be with the geometric series, but I still did not verify this case.

3c) n-compression

In general, a n-compression generates n new series as intermediate results.

ALTERNATING STABLE

Possibile.

DIVERGENT, CONSTANT TERMS

Irrelevant.

DIVERGENT, NON-CONSTANT TERMS, NOT PATTERNED

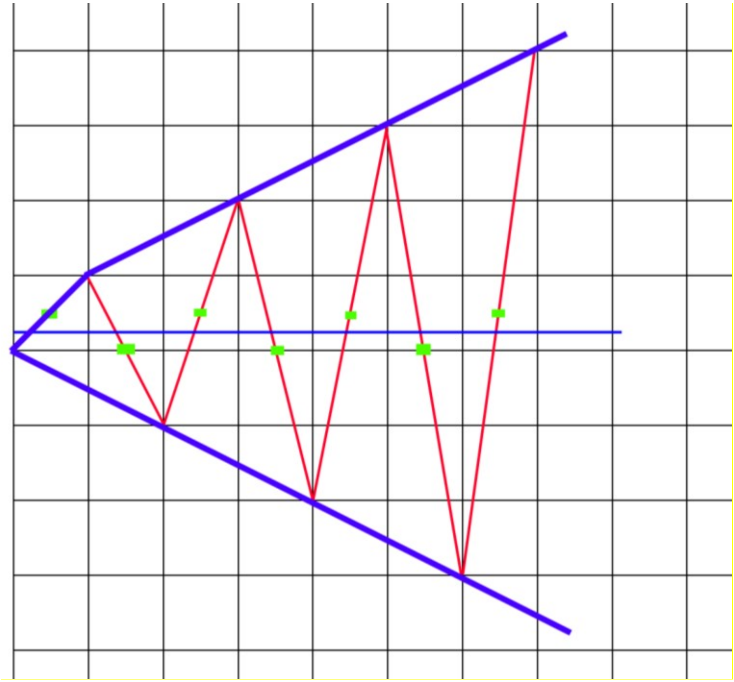
Not possible.

The compression by a factor which is not 2 seem to be very complicated and not very useful.

Extra

The compression seems to connect to the data compression topic.

For example, in the line graph of 1-2+3-4+...=1/4 the lines of the Up and Down series can be considered approximations with low resolution of the red graph: the resolution is lower and the inlets are not considered: the two lines approximates upper and lower the graph.



Extra

Sometimes the compression can give as one of the intermediate series a series with the same value of the starting series. This is a coincidence.

Numerically we need to pay attention and not confuse a partial resolution of a series via compression with the original series or with its associated value.

Attention

Example: the series 1-2+3-4+... must not be substituted with -1-1-1-1-... even if making the sum of the pairs we get this latter series!

This process is not allowed in non-convergent series, even though it might seem intuitive.

This is actually just a partial and approximated version of the real compression procedure described before.

We need to pay attention since it’s a very ambiguous choice in some cases.

Associative property applied to an infinity quantity of terms is not allowed in the infinite series.

More information about the properties of the infinite series in the 11th chapter.

4) Expansion

The expansion is the “reverse” of the compression.

Example: the expansion which is the reverse of the 2-compression is the 2-expantion.

If the series:

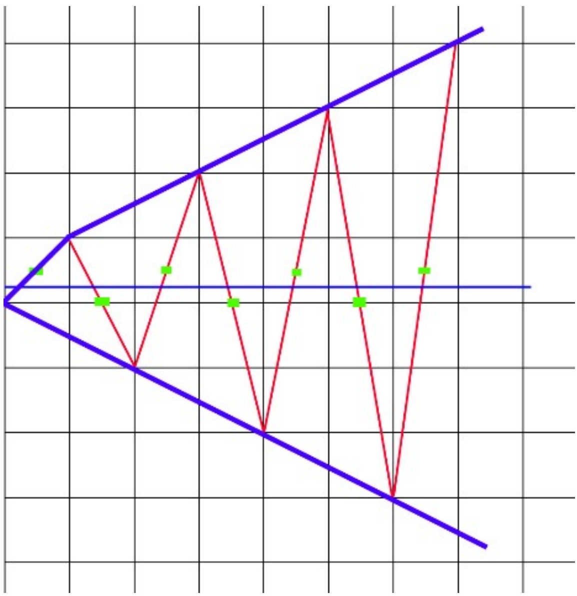
1-2+3-4+5-6+....

can be 2-compressed in two ways:

A: -1/2-1/2-1/2-....

B: 1+1/2+1/2+1/2+1/2

The series -1/2-1/2-1/2-... and the series 1+1/2+1/2+1/2+... can be expanded in 1-2+3-4+5-6+...



In red the series: 1-2+3-4+5-...

In blue, above, the series: +1+1/2+1/2+1/2+...

and in blue, at the bottom, the series:

-1/2-1/2-1/2-1/2-...

A n-compressed series gives as intermediate series n series, instead a series can be expanded in infinite ways.

Bidimensional expansion

Interesting is also the bidimensional expansion, which rewrite a series as a sum of more series (potentially an infinite quantity) in columns (vertically). We already saw the cases with a finite quantity, for example:

1+2+3+4+5+... →

(1+1+1+1+1+...+

 0+1+2+3+4+...)

Instead, with an infinite quantity of series there are problems (see 12th chapter for some examples).

At the moment I don’t know if this last type of expansion is really a 2-dimensional case of the expansion described in the beginning of this chapter, or if it’s another type.

5) The Add Problem

5a) The problem

The problem starts from a simple question:

1+2+2+2+2+... is the series 2+2+2+2+... with 1 added, or it's the series 2+2+2+... with 1 subtracted to the first 2?

Or it can start from another problem:

1+1+1+1+...=z

1+(1+1+1+...)=z

1+z=z

1=0 ABSURD

Thus, can we group a part of the series 1+1+1+1+...?

Superficially we can say that because valid operations are done and the result is absurd, thus the premises are not right which would mean the whole heuristic approach to the series is wrong.

Actually, if we keep studying these problems, we will notice that some rules/conditions have been not satisfied.

5b) Horizontal and vertical approaches and resolution

Let's see more in depth the problem for every type of series and if eventually it comes up.

We will see 2 approaches:

- horizontal
- vertical

The horizontal approach is the most intuitive and coincides with the vertical approach in the standard maths.

The vertical approach is the right approach in infinite series (where only sometimes the horizontal one works).

ALTERNATING STABLE

2-1+1-1+1-1+...=3/2

We know that 1-1+1-1+1-1+...=1/2

What is the value of 2-1+1-1+1-1+... ?

Horizontal approach:

2-1+1-1+... = 2-(1-1+1-...) = 2-1/2 = 3/2

Vertical approach:

1-1+1-1+... = 1/2 +

1+0+0+0+... = 1 =

2-1+1-1+... = 3/2

Vertical = Horizontal

The horizontal and vertical approaches bring to the same results in alternating stable series.

Another example:

0-1+1-1+... = ?

VERTICAL:

+1-1+1-1+... = 1/2   +

-1+0+0+0+... = -1   =

0-1+1-1+... = 1/2-1 = -1/2

0-1+1-1+...=-1/2

HORIZONTAL: 0-1/2 = -1/2

Coherent, hence: 0-1+1-1+...=-1/2

Another example:

We know that: 1-2+3-4+...=1/4

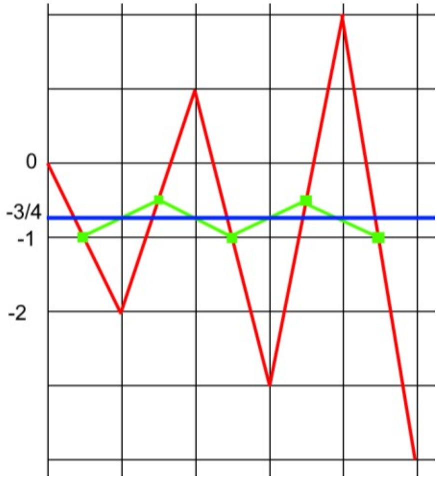
so, -1+2-3+4-...=-1/4

hence -1+1-1+...=-1/2

Which value is associated to -2+3-4+...=?

HORIZONTAL

We analyse the graph (we use the line graph since the series is stable): the extracted value is 3/4.



VERTICAL 1:

A: -1+2-3+4-...= -1/4 +

B: -1+1-1+1-...= -1/2 =

S: -2+3-4+5-...= -1/4-1/2 = -3/4

VERTICAL 2:

A: +1-2+3-4+5-6+... = 1/4 +

B: -1+0+0+0+0+0+... = -1 =

+0-2+3-4+5-6+... = 1/4 -1

0-2+3-4+5-6+... = -3/4

The series -2+3-4+5-6+... is stable hence -2+3-4+5-6+... has the same value of 0-2+3-4+5-6+...

-2+3-4+5-6+...=-3/4

Coherent.

DIVERGENT, CONSTANT TERMS

As we already saw, the criticism to infinite series resolved heuristically starts often from the criticism to the paradoxical situation caused by the grouping:

1+1+1+...=s

1+(1+1+...)=s

1+s=s

1=0 Absurd

This brings someone to believe the heuristic methods are not valid for infinite series.

Actually, some conditions and rules have not been satisfied (the right approach in infinite series is the vertical one and not the horizontal one).

Let's study the problem more in depth with an easier example: we know that

1+1+1+...=-1/2

2+1+1+1+... how much gives?

There are 2 possible approaches:

HORIZONTAL:

2+(1+1+1+...) = 2 -1/2 = 3/2

VERTICAL:

a: 1+1+1+... = -1/2 +

b: 1+0+0+... = 1 =

-1/2 + 1 = 1/2

2+1+1+1+1+...=1/2

Which of the 2 results is to consider right? 3/2 o 1/2?

The compression method helps us.

If we apply the compression to the series 1-2+3-4+... we can notice that a new series appears: it's half of the one we are studying:

U: 1 +1/2 +1/2 +1/2 +... = x

D: -1/2-1/2 -1/2 -1/2 -... = 1/4

We already know that the series 1-2+3-4+...=1/4 thus:

1/4 = (x + 1/4)/2

so, 1/4 is the average of the associated value to the U series and the associated value to the D series.

Therefore x=1/4 which means 1+1/2+1/2+1/2+...=1/4 thus

2+1+1+1+1+...=1/2

The vertical approach results valid and correct.

If we always use the horizontal approach, many contradictions and equivocation will come up, like for example:

1+1+1+1+... will be equivalent to both

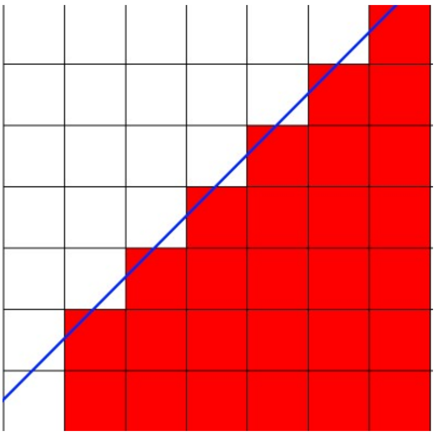
1+(1+1+1+...) = 1-1/2

and 1+1+(1+1+1+...) = 2-1/2

etc.

The horizontal approach is not consistent in case of divergent series with constant terms. Later we will see that the horizontal approach is inconsistent in other cases too.

Therefore, the vertical approach is right, and the horizontal approach is generally not valid.



Above a histogram graph which visualizes the series 2+1+1+1+...: the blue line intersects the y axis in 1/2 as a matter of fact 2+1+1+1+...=1/2

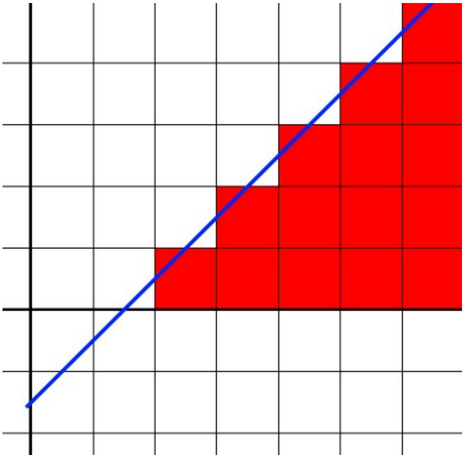
Similarly, 0+1+1+1+... = -3/2:

a: +1+1+1+1+1+...=-1/2 +

b: -1+0+0+0+0+...=-1 =

---

c: +0+1+1+1+1+...=-3/2 (where c=a+b)



In the graph, the straight line which cuts the steps, intersects the y axis in -3/2. This is another proof of the validity of the result.

(With the horizontal approach 0+1+1+1+... would have resulted -1/2 again).

We notice that the case 0+1+1+1+... coincides with the case of sliding of the divergent series with constant terms.

Formula for the addition (add problem) in the divergent series with constant terms

In general:

n+a+a+a+... = -a/2 + (n-a)

with n in whichever position

another example:

+0+0+1+1+1+... = -5/2

+1+1+1+1+1+... =-1/2 +

-1-1+0+0+0+... =-2 =

+0+0+1+1+1+... =-5/2

Generalizing sliding for divergent series with constant terms

In the light of the resolution of the add problem, the sliding for divergent series with constant terms can be generalized in this way:

Let s: 1+1+1+...=-1/2 . It slid of n terms is:

0+...+0+1+1+1+... (with n initial 0s):

$s^{(n)}$: 0+...+0+1+1+1+... = -1/2-n

And in general, with s: a+a+a+...=r=-a/2 , slid of n terms is:

0+...+0+a+a+a+... (with n initial 0s):

$s^{(n)}$: 0+...+0+a+a+a+...=r-na =-a/2-na

Another example (non-consecutive exceptions terms)

+0+1+0+1+1+... = -5/2

+1+1+1+1+1+... =-1/2 +

-1+0-1+0+0+... =-2 =

+0+1+0+1+1+... =-5/2

+0+1-2+1+1+... = -9/2

+1+1+1+1+1+... =-1/2 +

-1+0-3+0+0+... =-4 =

+0+1-2+1+1+... =-9/2

DIVERGENT, NON-CONSTANT TERMS, NOT PATTERNED

Knowing that 1+2+3+4+...=-1/12

2+3+4+...=-7/12

The problem can be solved via vertical approach like in the previous cases.

A: 1+2+3+4+5+...=-1/12 +

B: 1+1+1+1+1+...=-1/2 =

C: 2+3+4+5+6+...= -1/12-1/2 = -7/12

with C=A+B

With the horizontal approach we would have said that 2+3+4+... = -1/12-1, but this approach is inconsistent in many other occasions, as a matter of fact, it often gives contradictory results.

Another example:

0+1+2+3+4+...=5/12

A: +1+2+3+4+5+...=-1/12 +

B: -1-1-1-1-1-...=1/2 =

C: 0+1+2+3+4+...=-1/12+1/2=5/12

with C=A+B

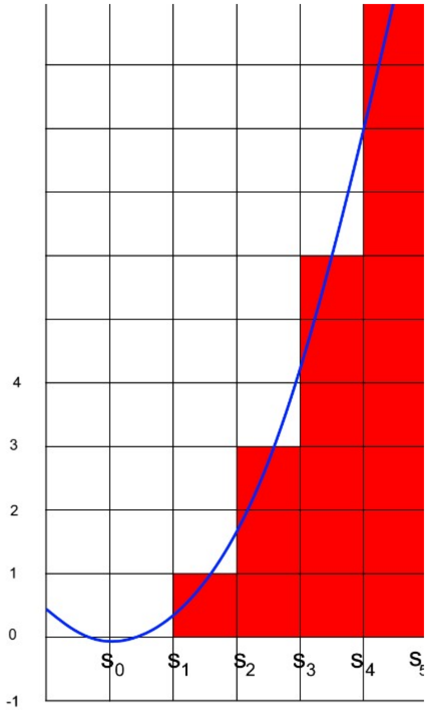
It’s very counterintuitive the fact that adding a 0 at the start of a divergent series the result changes.

If we think the = sign or the sum in these heuristic situations have not the standard meaning, thus the problem does not arise, at least not much.

Visualization of the 5/12

If we study the function of the partial sums and the summarizing line of the graph of the partial sums, we notice that the latter is a parabola with the vertex on the y axis

For x<0 the curve is decrescent.



In x=0 the blue curve has ordinate of approximately 0.5, value near 5/12, which we got previously numerically.

The graph without the first column is the identical to the graph of 1+2+3+4+... (see chapter 1, paragraph: ‘famous series’)

The illustration is not precise because it has been done with a simple drawing software and not with a maths software.

Moreover, it's not clear to me the procedure to find the curve, like in the case of the graph of 1+2+3+4+..., the curve is qualitative.

We can get a representation of the value 5/12 thanks to the graph of the function 0+1+2+3+4+...

Moreover 0+1+2+3+4+5+...=5/12 shows consistence and coherence when used in operations with other series.

The infinite series heuristically treated are the reflection of the function of the partial sums of the series.

As a matter of fact, the function of the partial sums of the series 1+2+3+4+... is a parabola: $y=x^2/2+x/2$

More information about the associated functions in the 8th chapter.

0+0+1+2+3+4+5+...=23/12

The procedure is similar to the previous ones.

+1+2+3+4+5+6+...=-1/12 +

-1-2-2-2-2-2-....= 2 =

0+0+1+2+3+4+...=-1/12+2=23/12

Note that -1-2-2-2-... is:

-(1+2+2+2+...) and 1+2+2+2+... gives the value: -2/2-1 = -2

Therefore -1-2-2-2-... = 2

Therefore 0+0+1+2+3+4+...=23/12

The visualization of the result in the graph + parabola is similar to the one of 0+1+2+3+4+.... (Warning: in the graph shown above, the curve does not intersect the point 23/12, but it's close to it, since the graph is not precise, I'm sorry for the approximation).

At the moment a general formula for the add problem s not available for the divergent series with non-constant terms.

The sliding of geometric series is interesting to study.

For example, how much does 0+2+4+8+16+... give?

2+4+8+16+...=-2 and 0+2+4+8+16+...=-2 too.

Sliding a geometric series does not change its value

We saw this in the 2nd chapter: sliding

Example:

+2+4+8+16+32+... = -2 +

-2-2-4-8 -16+... = 0 =

0+2+4+8 +16+... = -2

5c) More important examples

0+2+3+4+5+...=-13/12

+1+2+3+4+5+...=-1/12 +

-1+0+0+0+0+...=-1 =

0+2+3+4+5+...=-13/12

1+3+5+7+9+... = 1/3

a: +1+2+3+4+5+... = -1/12 +

b: -1-1-1-1-1-... = 1/2 =

---

c: +0+1+2+3+4+... = 5/12 (c=a+b)

a: +1+2+3+4+5+... = -1/12 +

c: +0+1+2+3+4+... = 5/12 =

---

d: +1+3+5+7+9+... = 4/12 = 1/3 (d=a+c)

The formula for the partial sums of 1+3+5+7+... is:

$$y=x^2$$

as a matter of fact:

$s_0=0$

$s_1=1$

$s_2=1+3=4$

$s_3=1+3+5=9$

$s_4=1+3+5+7=16$

...

-1+0+1+2+3+4+5+... =11/12

+1+2+3+4+5+...=-1/12

-2-2-2-2-2-...=1

-1+0+1+2+3+...=-1/12+1=11/12

-2-1+0+1+2+3+4+...=17/12

+1+2+3+4+5+...=-1/12

-3-3-3-3-3-...=3/2

-2-1+0+1+2+...=17/12

6) Dilation

The dilation is not the reverse operation of the compression, even though it seems to be it.

A dilation is for a n integer positive factor (n-dilation).

The 2-dilation is very important.

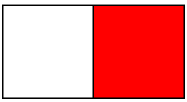
6a) 2-dilation

The 2-dilation is the dilation for the factor 2.

There are 2 main types of 2-dilation: 0x and x0, and one more type: flat/symmetric dilation.

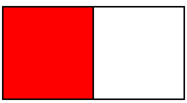
0x Dilation

a+b+c+d+... → 0+a+0+b+0+c+0+d+...



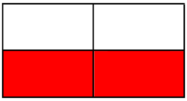
x0 Dilation

a+b+c+d+... → a+0+b+0+c+0+d+0+...



flat/symmetric Dilation

a+b+c+d+... → a/2+a/2+b/2+b/2+...



In the names (0x, x0) x indicates a term from the original series.

0x 2-dilation

ALTERNATING STABLE

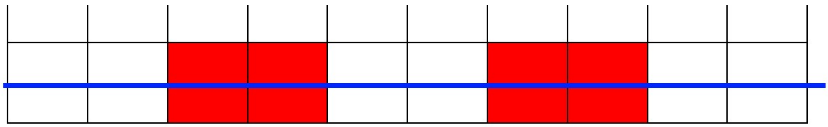
0x dilating an alternating stable series does not change its value.

Example:

1-1+1-1+... =1/2

0+1+0-1+0+1+0-1+... = 1/2

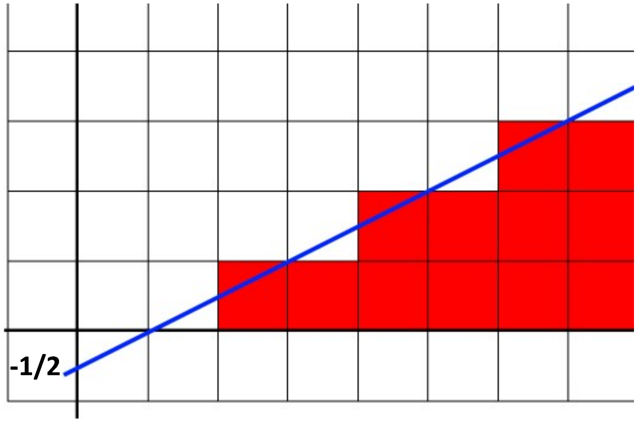
Graphic intuition



DIVERGENT, CONSTANT TERMS

We know that 1+1+1+... = -1/2

0+1+0+1+0+1+... = ?



(Note: we have to remember that there is always a null first step (first empty column in the histogram above) like in the graph of 1+1+1+...)

0+1+0+1+0+1+...=-1/2

If we squeeze the graph horizontally for a factor 2, we get the same graph of 1+1+1+...=-1/2. So, if we have 2 identical graphs, they must have the same properties, like that of the summarizing line, thus: the graph of 0+1+0+1+... has the same summarizing line of 1+1+1+... which intersects the y axis in -1/2.

1+1+1+...=-1/2 and 0+1+0+1+0+1+...=-1/2

We can also find the result via compression (see more details in the dedicated chapter)

X: 0 +1 +0 +1 +...

U: 0 +1/2 +1/2 +1/2 +...

D: 1/2 +1/2 +1/2 +1/2 +...

U+D: 1/2 +1 +1 +1 + ...

(U+D)/2: 1/4 +1/2 +1/2 +... = -1/4 -1/4 = -1/2

Coherent.

In general:

A divergent series with constant terms, 0x dilated has the same value of the original series.

0+a+0+a+0+a+...=-a/2

Now the "demonstration" of 1+1+1+...=-1/2 which starts from:

1+1+1+1+...=s

2+2+2+2+...=2s

-2-2-2-2-...=-2s

0-2+0-2+0-2+...=-2s

hence

1+1+1+1+...=s

0-2+0-2+...=-2s (is always -2s)

1-1+1-1+...=-s

s=-1/2

Has more sense.

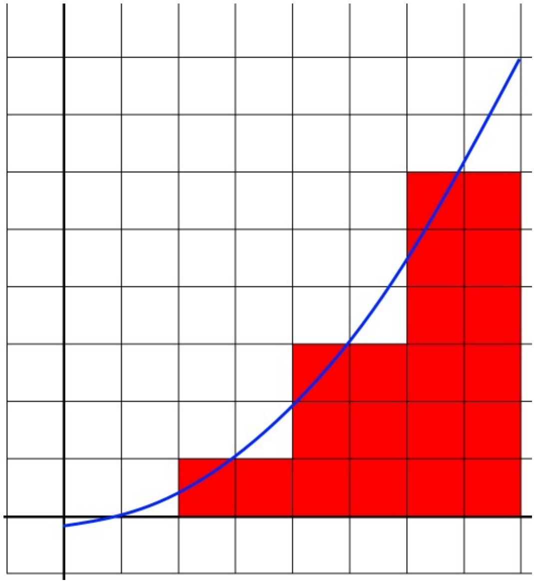
We notice that the series 0+1+0+1+0+1+... is the series 1+0+1+0+... slid of 1.

Similarly, the series 0+a+0+a+... is a+0+a+0+... 1-slid.

DIVERGENT, NON-CONSTANT TERMS, NOT PATTERNED

We know that 1+2+3+4+...=-1/12

0+1+0+2+0+3+... = ?



If we squeeze the graph horizontally of a factor 2 we get the graph of 1+2+3+4+...

Therefore, it's absurd if the same graph has 2 different summarizing line, therefore also 0+1+0+2+0+3+...=-1/12

A divergent series with non-constant terms, 0x dilated, has the same value of the original series.

The demonstration of Ramanujan, for -1/12, makes sense now.

It starts from:

1+2+3+4+5+6+... = s

-4-8-12-16-... = 4s

-4s = 0-4+0-8+0-16+...

The 0x dilation does not change the value of a series,

1+2+3+4+5+6+... = s +

0-4+0-8+0-16+...=-4s (is -4s here too) =

1-2+3-4+5-6+... =-3s

1/4 = -3s

s=-1/12

Coherent.

We also notice that the series 0+1+0+2+0+3+... is the series 1+0+2+0+3+0+... slid of 1.

And in general, a series $0+x_1+0+x_2+...$ is the series $x_1+0+x_2+0+...$ slid of 1.

x0 2-dilation

ALTERNATING STABLE

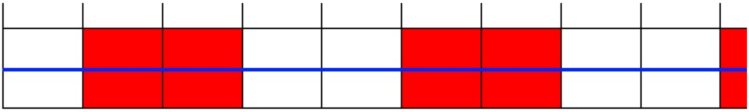
An alternating stable series does not change value if x0 dilated.

Example:

1-1+1-1+...=1/2

1+0-1+0+1+0-1+0+... = 1/2

Graphic intuition



the intersection is always in the ordinate 1/2.

DIVERGENT, CONSTANT TERMS

Let's analyse the case for:

1+0+1+0+1+0+...

At first glance, seems that it's equivalent to 0+1+0+1+...=-1/2, but it's not.

1+0+1+0+1+0+... = 0

Let's look at the graph:

The blue line intersects the y axis in 0.

As a matter of fact, via compression:

X: 1 +0 +1 +0 +...

U: 1 +1/2 +1/2 +1/2 +... +

D: 1/2 +1/2 +1/2 +1/2 +... =

D+U: 3/2 +1 +1 +1 +...

(D+U)/2: 3/4 +1/2 +1/2 +1/2 +... = -1/4+1/4 = 0

Moreover, the result can be obtained also from:

A: 1+1+1+1+1+1+...=-1/2 +

B: 0+1+0+1+0+1+...=-1/2 =

C: 1+0+1+0+1+0+...=0

Where C=A-B

Thus 1+0+1+0+1+0+...=0 and

In general: a+0+a+0+a+0+...=0

So, even the disposition affects the result or associated value (0+1+0+1+... has not the same value of 1+0+1+0+... even though they have the same summed terms; see commutative property in the 11th chapter: model)

We notice that the series 0+1+0+1+0+1+... is the series 1+0+1+0+1+0+... slid of 1.

+1+0+1+0+1+0+...=0

-1+1-1+1-1+1-...=-1/2

0+1+0+1+0+1+...=-1/2

DIVERGENT, NON-CONSTANT TERMS, NOT PATTERNED

1+0+2+0+3+0+4+0+...=1/24

Graphically it's not easy to mentally realize this result, but it's easy to calculate.

Numerical explanation

1+0+2+0+3+0+4+0+...=s

2+0+4+0+6+0+8+0+...=2s

+1+3+5+7+9+... = 1/3 +

+2+2+2+2+2+... = -1 =

+3+5+7+9+11+...= 1/3 – 1 = -2/3

3+5+7+9+11+...=-2/3

0+3+0+5+0+7+...=-2/3 (0x dilation)

0-3+0-5+0-7+...=2/3

2+0+4+0+6+0+8+0+...=2s +

0-3+0-5+0-7+...=2/3 =

2-3+4-5+6-7+...=2s+2/3

-2+3-4+5-6+7-8+...=-2s-2/3

Due to stability of -2+3-4+5-6+... :

1+(-2+3-4+...) = (-2s-2/3)+1

1/4 = -2s-2/3+1

1/4 +2/3-1 = -2s

-1/12 = -2s

s= 1/24

So

1+0+2+0+3+0+4+0+ ... = 1/24

1+0+3+0+5+0+7+0+...=1/12

This series must not be confused with the series 1+3+5+7+...

The value of this series is very easy to calculate:

1+2+3+4+5+...=-1/12

2+4+6+8+12+... = -1/6

0+2+0+4+0+6+... = -1/6 (0x dilation)

0-2+0-4+0-6+... = 1/6

hence

a: 1+2+3+4+5+6+...=-1/12 +

b: 0-2+0-4+0-6+... = 1/6 =

---

c: 1+0+3+0+5+0+... = -1/12 + 1/6 = 1/12

where c=a+b

Therefore 1+0+3+0+5+0+... = 1/12

At the moment there isn't any quick method to identify a value of a series 0x dilated starting from a divergent series with non-constant terms, not patterned, and using heuristic direct manipulations of the series, instead is very easy to use the associated functions, see chapter 9.

We notice that the series 1+0+2+0+3+0+... is the series 0+1+0+2+0+3+... slid of -1; and in general a series $x_1+0+x_2+0+x_3+0+\dots$ is $0+x_1+0+x_2+0+x_3+0+\dots$ slid of -1.

Symmetric/flat 2-dilation

ALTERNATING STABLE

Example:

1-1+1-1+... dilated symmetrically becomes: 1/2+1/2-1/2-1/2+...

The series can be calculated as the sum of the relative 0x and x0 dilated divided by 2.

0 +1/2 +0 -1/2 +0 +1/2 +0 -1/2 +...=1/4 +

1/2 +0-1/2 +0 +1/2 +0 -1/2 +0 +...=1/4 =

1/2+1/2-1/2-1/2+1/2+1/2-1/2-1/2+...=1/2

The value is equal to the value of the starting series

in general:

Let A be a stable series: $x_1+x_2+x_3+x_4+...$ = s (the dilatated 0x and x0 have the same value)

1/2 $(0+x_1+0+x_2+0+x_3+0+x_4+...)$ = 1/2*s +

1/2 $(x_1+0+x_2+0+x_3+0+x_4+0+...)$ = 1/2*s =

---

1 $(x_1+x_1+x_2+x_2+x_3+x_4+x_4+...)$ = 1s

$x_1+x_1+x_2+x_2+x_3+x_4+x_4+...=s$

DIVERGENT, CONSTANT TERMS

a+a+a+... → a/2 +a/2 +a/2 +a/2 +...

The series has half value of the value of the starting series

DIVERGENT, NON-CONSTANT TERMS, NOT PATTERNED

It’s not known a simple method to detect the value (indeed, the series depends on the x0 dilation of a divergent series with non-constant terms)

6b) 3-dilation

A series can be dilated of a factor 2, as well as an integer positive n factor like for example 3.

The 3-dilation is a dilation for a factor 3, hence there will be 3 main types of 3-dilated series: x00, 0x0, 00x (where x represents a term from the original series), moreover we can find the flat-dilated.

Example:

a+b+c+d+... →

00x) 0+0+a+0+0+b+0+0+c+...

0x0) 0+a+0+0+b+0+0+c+0+...

00x) 0+0+a+0+0+b+0+0+c+...

flat) a/3+a/3+a/3+b/3+b/3+b/3+c/3+c/3+c/3+...

00x 3-dilation

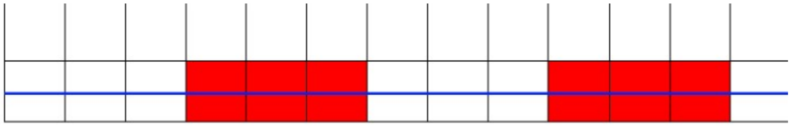
ALTERNATING STABLE

Like in the 2-dilation 0x, a 3-dilation 00x in an alternating stable series does not change the value of the series.

1-1+1-1+... = 1/2 →

→ 0+0+1+0+0-1+0+0+1+0+0-1+...=1/2

Graphic illustration



The blue line intersects the y axis in 1/2 like the series 1-1+1-1+...=1/2

DIVERGENT, CONSTANT TERMS

0+0+1+0+0+1+...=-1/2

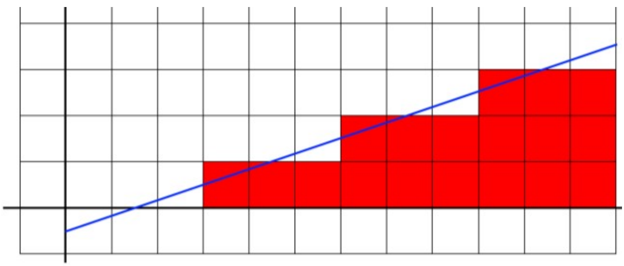
0+0+a+0+0+a+...=-a/2

Similarly, to the explanation of the 2-dilation 0x, the graph of a series 00x dilated, horizontally squeezed for a factor 2, becomes the graph of a+a+a+... thus both the series must have the same value or an absurd comes up.

The 3-dilation 00x does not change the value of a series with constant terms

Later we will see a general formula for the n-dilation of the type 0...x

Graphic illustration



The blue line intersects in -1/2 the y axis.

The 3-compression can be applied to the series 0+0+1+0+0+1+... to find its value with success.

DIVERGENT, NON CONSTANT TERMS, NOT PATTERNED

Similarly, to the explanation for the 2-dilation 0x, the graph of a 00x dilated series horizontally squeezed for a factor 3, becomes the graph of a+b+c+... therefore both the series must have the same value or an absurd comes up.

The 3-dilation 00x does not change the value of a divergent series with non-constant terms.

0x0 3-dilation

ALTERNATING STABLE

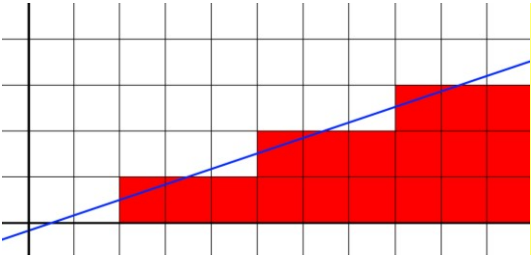
The value is not changed. Similar motivations to those reported in the 2-dilation for stable series.

DIVERGENT, CONSTANT TERMS

Example:

1+1+1+1+... → 0+1+0+0+1+0+0+1+0+...=-1/6

Graphic illustration



This graph is not very accurate, but we can geometrically calculate that the blue straight line intersects the y axis in -1/6.

in general: 0+a+0+0+a+0+...=-a/6

DIVERGENT, NON-CONSTANT TERMS, NOT PATTERNED

At the moment, a procedure which uses directly the manipulation of the series terms to find the value of the series of this type is not known.

We need to use the associated functions (see chapter 9).

Thanks to the associated functions to the partial sums we can find further results.

Like for example:

0+1+0+0+2+0+0+3+0+...=-1/36

x00 3-dilation

ALTERNATING STABLE

The value is not changed just like in the 2-dilation.

DIVERGENT, CONSTANT TERMS

1+1+1+... → 1+0+0+1+0+0+...=1/6

If we analyse geometrically the graph, we notice that the blue straight line intersects the y axis in 1/6.

DIVERGENT, NON-CONSTANT TERMS, NOT PATTERNED

At the moment I don't know any procedure to directly manipulate the series in these cases to find the associated value.

We need to use the associated functions (see chapter 9):

1+0+0+2+0+0+3+0+0+...=5/36

Flat 3-dilation

The flat 3-dilation is similar to the flat/symmetric 2-dilation. The explanation is omitted for brevity.

6c) n-dilation and general formula

ALTERNATING STABLE

Every n-dilation in an alternating stable series does not change the value of the series

So, a dilation of the type 0x, x0, 00x, 0x0, 000x, 000x00, ... does not change the value of an alternating stable series.

DIVERGENT, CONSTANT TERMS

Considering as example the 3-dilation and comparing every result for each case, we notice that the values are in function of the position p of the term among the 0s.

We can find a general formula for the values of the n-dilated series with n integer positive factor (1, 2, 3, 4, ...):

Starting from 1+1+1+1+...=-1/2 ,

its d-dilated, where the non-0 term is positioned among the 0s in the position p, gives value:

$$y = -\frac{1}{2} + \frac{d-p}{d}$$

Examples with d and p:

Dilating the series 1+1+1+...=-1/2

0+0+1+0+0+1+... = -1/2

0+0+1+0+0+1+...   (d=3, p=3)

0+0+1+0+0+1+...=-1/2+(3-3)/3 = -1/2

0+1+0+0+1+0+...=-1/6

0+1+0+0+1+0+...   (d=3, p=2)

0+1+0+0+1+0+...=-1/2+(3-2)/3=-1/6

0+1+0+1+0+1+ ...=-1/2

0+1+0+1+0+1+...   (d=2, p=2)

0+1+0+1+0+1+ ...=-1/2+(2-2)/2 = -1/2

1+0+0+1+0+0+... = 1/6

1+0+0+1+0+0+... (d=3, p=1)

1+0+0+1+0+0+... = -1/2+(3-1)/3 = 1/6

0+0+0+1+0+0+0+1+...=-1/2

0+0+0+1+0+0+0+1+... (d=4, p=4)

0+0+0+1+0+0+0+1+...=-1/2+(4-4)/3 = -1/2

0+1+0+0+0+1+0+0+... = 0

0+1+0+0+0+1+0+0+...   (d=4, p=2)

0+1+0+0+0+1+0+0+... = -1/2+(4-2)/4=0

1+1+1+1+...=-1/2

1+1+1+1+...   (d=1, p=1)

1+1+1+...=-1/2+(1-1)/1 =-1/2

Note:

0+1+0+0+0+1+0+0+...=0

0+0+0+1+0+0+0+1+...=-1/2

0+1+0+1+0+1+0+1+...=-1/2

Coherent in both members.

And:

1+0+0+1+0+0+... = 1/6

0+1+0+0+1+0+... = -1/6

0+0+1+0+0+1+... = -1/2

1+1+1+1+1+1+... = -1/2

Coherent in both members.

And in general, for a series:

a+a+a+a+... =-a/2

The dilated for a factor d, with the term a at the position p among the 0s (p from 1 to d) gives value:

$$y = -\frac{a}{2} + \frac{a(d-p)}{d}$$

0+4+0+0+4+0+... = -2/3

4+4+4+... dilated with (d=3, p=2):

0+4+0+0+4+0+... = -4/2+4(3-2)/3 = -2/3

DIVERGENT, NON-CONSTANT TERM, NOT PATTERNED

It's not available at the moment a general method via direct manipulation of the series terms to d-dilate a series of this type (with p position of the term among the 0s).

We have to use numerical "demonstrations", which are usually not easy, or the associated functions (see chapter 9).

Extra

The dilation of the type 0....x (x last position) corresponds to the dilation via associated functions, while the other types (non 0...x) are in associated functions: dilatation + translation (sliding).

Dilation^n

The dilation can be applied more times (with "dilation to the n" we mean applying the dilation n times)

If the series 1+1+1+1+... is x0 dilated and then 0x dilated, we get:

1+1+1+1+... → 1+0+1+0+... → 0+1+0+0+0+1+0+0+...

We get a 0x00 dilated of 1+1+1+...

x0, then x0, we get a x000

0x, then 0x, we get a 000x

To find the name of the resulting dilation, we need to apply the dilation to the name of the dilations applied to the series.

We can also easily generalize the application of a dilation n times with the associated functions (see 9th chapter).

6d) More important examples

Thanks to the dilation formula we can see many of the previous series in a new perspective:

1-2+3-4+5-6+... = 1/4

1+0+3+0+5+0+... = 1/12 (already seen)

-2-4-6-8-12-... = -2(1+2+3+...) =1/6

0-2+0-4+0-6+...=1/6 (same value, since it's 0x dilated)

1+0+3+0+5+0+... = 1/12 +

0-2+0-4+0-6+... = 1/6 =

1-2+3-4+5-6+... = 1/12+1/6 = 3/12 = 1/4

Coherent.

Or

1-1+1-1+...=1/2

1+0+1+0+...=0 +

0-1+0-1+... = 1/2 =

1-1+1-1+... = 1/2

Coherent in both members.

In this last case, we can indicate the sum of the two series as a “comb sum”, which means that the two series are complementary: where there are 0s in the first series, in the second there are non-0 terms, and where there are non-0 terms in the first, there are 0s in the second.

We will indicate with comb sum also a vertical sum of two series where the second is dilated but the first is not.

7) Patterned series

Patterned series are very important:

s: a+b+c+...+n+a+b+c+...+n+...

A patterned series (general description above) is a series where some terms are periodically summed, and the terms are arranged in the same order every time.

In the previous description the terms go from a to n (n generical term)

There are a lot of sub-types of this series, so we will show different example without a classification.

Remember also that a dilated series from a series x+x+x+... has value -x/2+x(d-p)/d

1-2+1-2+1-2+...= 1

Pattern: (1, -2)

a: 1+0+1+0+1+0+...=0 +

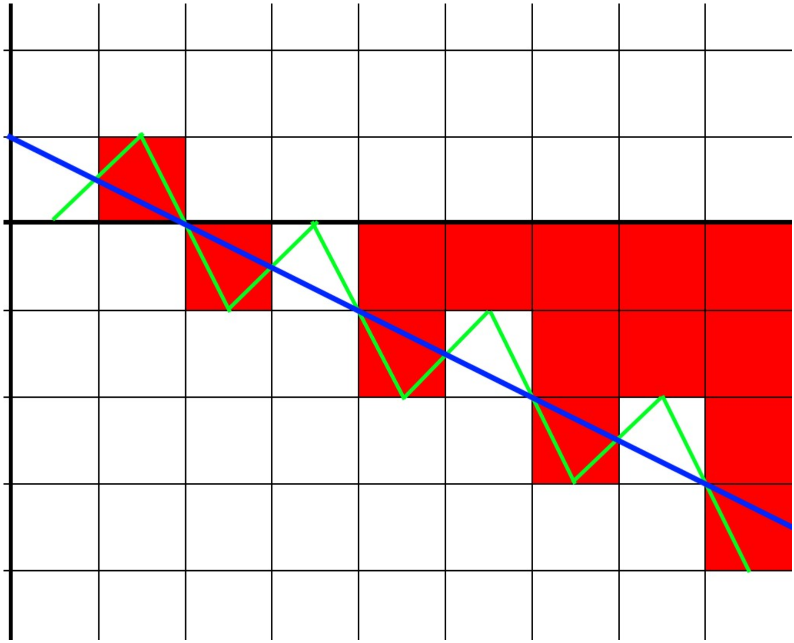
b: 0-2+0-2+0-2+...=1 =

---

c: 1-2+1-2+1-2+...=1 (c=a+b)

This series is alternating but not stable, and it’s divergent, as a matter of fact the stability line is not parallel to the x axis and it’s decreasing.

Graph:



In the histogram the blue line intersects the y axis in the ordinate 1.

Remember that the line graph is not good to extract the value, and we used the histogram, indeed (more information in the 10th chapter)

Thus, we can generalize a patterned series with 2 repeating terms in this way:

a+b+a+b+a+b+...=-b/2

a: a+0+a+0+...=0 +

b: 0+b+0+b+...=-b/2 =

---

c: a+b+a+b+...=-b/2 (c=a+b)

An important question is: can we apply here the 2-compression?

2-compression:

1-2+1-2+1-2+...

```
U:        -1/2 -1/2 -1/2 -...       +
D:         1   -1/2 -1/2 -...       =
-----------------------------------------
(U+D):     1/2 -1    -1   -...
(U+D)/2:   1/4 -1/2  -1/2 -...
```

1/4 -1/2 -1/2 -... = -(-1/2)/2 + (1/4 -(-1/2)) = 1

Coherent.

Another method for the resolution

Using the 0x dilation:

```
a: 1+1+1+1+1+1+... =-1/2     +
b: 0-3+0-3+0-3+... = 3/2     =
-----------------------------------------
c: 1-2+1-2+1-2+... =2/2=1
```

Coherent.

(0-3+0-3+0-3+... is the 0x dilated of -3-3-3-... , thus they have the same value which is 3/2)

Thanks to the formula of the dilation is possible to find the value for more “artistic” series too, like:

1+2+3+1+2+3+1+2+3+... = -5/3

1+0+0+1+0+0+1+0+0+... = 1/6 +

0+2+0+0+2+0+0+2+0+... = -1/3 +

0+0+3+0+0+3+0+0+3+... = -3/2 =

1+2+3+1+2+3+1+2+3+... = -5/3

a+b+c+a+b+c+a+b+c+... = (a-b-3c)/6

a+0+0+a+0+0+a+0+0+...=+a/6 +

0+b+0+0+b+0+0+b+0+...=-b/6 +

0+0+c+0+0+c+0+0+c+...=-c/2 =

---

a+b+c+a+b+c+a+b+c+...=(a-b-3c)/6

We can also study a generic patterned series like:

$x_1+x_2+x_3+...+x_n+x_1+x_2+x_3+...+x_n+...$

(with pattern: $x_1+x_2+x_3+...+x_n$)

Like before, to find the value of this series, we have to considerate it as a sum of different constant terms series (each series has its own term: x_1, x_2, x_3, ..., x_n) every series is dilated of n (where n is the number of elements in the pattern) and has the original term in the position p (where p corresponds to the position of the term in the pattern: a number from 1 to n):

x_1 +0 +0 +... +0 +... +

0 +x_2 +0 +... +0 +... +

0 +0 +x_3 +... +0 +... +

... +

0 +0 +0 +... +x_n+... =

---

x_1 +x_2 +x_3 +... +x_n +...

Because each of these series has a value:

-x/2+x(d-p)/d

with d=n, x goes from x_1 to x_n and p goes from 1 to n (where n is the number of terms in the pattern)

Thus, we just need to use the formula:

$x_1+x_2+x_3+...+x_n+x_1+x_2+x_3+...+x_n+...=s$

$$s=\sum_{i=1}^{n}\left(-\frac{x_i}{2}+\frac{x_i(n-i)}{n}\right)$$

which can be written as:

$x_1+x_2+x_3+...+x_n+x_1+x_2+x_3+...+x_n+...=s$

$$s=\frac{1}{2}\sum_{i=1}^{n} x_i-\frac{1}{n}\sum_{i=1}^{n} x_i i$$

or

$$s=\frac{x_1(n-2)+x_2(n-4)+x_3(n-6)+\cdots+x_n(n-2n)}{2n}$$

or

$$s=\frac{1}{2n}\sum_{i=1}^{n} x_i\,(n-2i)$$

Let’s calculate a few series using this formula:

1+1+1+1+... (N=1, pattern: 1)

s=(1/2)*(1)-(1/1)*(1*1)=-1/2 coherent

1-1+1-1+... (N=2, pattern 1-1)

s=(1/2)*(1-1)-(1/2)*(1*1-1*2)

s=0-(1/2)*(-1)=1/2 coherent

1+2+3+1+2+3+... (N=3, pattern: 1+2+3)

s=(1/2)*(1+2+3)-(1/3)(1*1+2*2+3*3)

s=6/2-14/3=(18-28)/6=-10/6=-5/3 coherent.

8) Riemann Zeta

The Riemann Zeta is a very famous function. On the internet you can find a lot of information about it.

It's connected to the infinite series.

WARNING! We will talk about this function in a non-rigorous way, and from a mathematical point of view not always correct (many times it will be heuristic).

We will indicate the Riemann Zeta function with the letter Z.

The Riemann Zeta is defined as:

$$Z(t) = \frac{1}{1^t} + \frac{1}{2^t} + \frac{1}{3^t} + \frac{1}{4^t} + \cdots$$

with t complex number.

Here we will considerate only the cases with t real number.

Let t=-x , so we can use a simpler function but still connected to the Zeta di Riemann:

$$S(x)=1^x+2^x+3^x+4^x+...$$

Where S(x)=Z(-t)

Let's focus on S(x) now:

S(0) = 1+1+1+1+... = -1/2

S(1) = 1+2+3+4+5+...= -1/12

S(2) = 1+4+9+16+25+... = 0

We already saw these three series in the previous chapters.

S(-1)= 1+1/2+1/3+1/4+... diverges and hasn't any associated value.

Many other values have been found.

Let's analyse the situation for x>=0

S(0) = 1+1+1+1+... can easily be resolved via "comb sum" (we already saw it previously):

+1+1+1+1+...= y   +

-0-2-0-2+...= -2y   =   (-2-2-2-... 0x dil.)

+1-1+1-1+...= -y

But 1-1+1-1+...=1/2 hence:

1+1+1+...=-1/2

To recap: we did y-2y.

Similarly, for 1+2+3+4+...=-1/12

"Demonstration" of Ramanujan (we already saw it previously)

+1+2+3+4+5+6+... =y   +

-0-4-0-8-0-12-...=-4y   =   (-4-8-12-... 0x dil.)

+1-2+3-4+...=-3y

1/4=-3y

y=-1/12

Thus, we did: y-4y.

1+4+9+16+25+36+...=y +

0-8+0-32+0+-72+...=-8y = (-8-32-72-... dil.)

1-4+9-16+25-...=-7y

0=-7y

y=0

Thus, we did: y-7y

In general, we can notice that, given a S(x) (where x is the exponent) the subtraction S(x)-(2^{x+1})S(x) (with (2^{x+1})S(x) 0x dilated) brings to the alternating version of S(x) which we will call T(x).

1+1+1+1+... → y-2y=-y → 1-1+1-1+... (x=0 → 2^{0+1}=2)

1+2+3+4+... → y-4y=-3y → 1-2+3-4+... (x=1 → 2^{1+1}=4)

1+4+9+16+... → y-4y=-7y → 1-4+9-16+... (x=2 → 2^{2+1}=8)

In general, we notice that to go from the series S(x) to its alternating (which we call T(x)) we just need to make the subtraction in columns (vertical) of S(x) and the 0x dilated of ($2^{(x+1)}$)S(x):

$$S(x) \rightarrow S(x) - 2^{x+1}S(x) \rightarrow T(x)$$

where

$$S(x) = 1^x + 2^x + 3^x + 4^x + \cdots$$

and

$$T(x) = 1^x - 2^x + 3^x - 4^x + \cdots$$

The problem to find the value of S(x) is moved to the problem of finding the value of T(x).

But how to find the value of T(x)?

The problem has been studied in the section of the sliding sum where we saw that in the cases of stable series, we just need to apply some sliding sums until the result will be finite.

At the moment, I don't know if the alternating series relative to the Riemann Zeta series are always stable, but I suppose they are.

Example:

```
1-4+9-16+25-36+...     +

0+1-4+9 -16+25-...     =
---------------------------
1-3+5-7 +9 -11+...     +

0+1-3+5 -7 +9 -...     =
---------------------------
1-2+2-2 +2 -2 +...     +

0+1-2+2 -2 +2 -...     =
---------------------------
1-1+0+0 +0 +0 +...     =
---------------------------
0
```

The sliding sum has been applied 3 times.

The different sums can be also represented in this way:

1 -4 +9 -16+25 -36 +... = s

0 +1 -4 +9 -16 +25 -... = s

0 +1 -4 +9 -16 +25 -... = s

0 +0 +1 -4 +9 -16 +... = s

0 +1 -4 +9 -16 +25 -... = s

0 +0 +1 -4 +9 -16 +... = s

0 +0 +1 -4 +9 -16 +... = s

0 +0 +0 +1 -4 +9 +... = s

---

1 -1 +0 +0 +0 +0 +... = 8s

And we can rewrite the whole thing in the following way: (depending on how many 0 there are in the beginning of a series):

1 -4 +9 -16+25 -36 +... = s

0 +1 -4 +9 -16 +25 -... = s

0 +1 -4 +9 -16 +25 -... = s

0 +1 -4 +9 -16 +25 -... = s

0 +0 +1 -4 +9 -16 +... = s

0 +0 +1 -4 +9 -16 +... = s

0 +0 +1 -4 +9 -16 +... = s

0 +0 +0 +1 -4 +9 +... = s

---

1 -1 +0 +0 +0 +0 +...

We can then merge all with coefficient per each type of slid series:

1 x (1 -4 +9 -16 +25 -36 +...)

3 x (0 +1 -4 +9   -16 +25 -...)

3 x (0 +0 +1 -4   +9 -16 +...)

1 x (0 +0 +0 +1   -4 +9 +...)

---

1 -1 +0 +0   +0 +0 +...

Analysing the similar processes for the series:

1-1+1-1+...

1-2+3-4+...

we can notice that:

the finite sum of all slid alternating series is equal to (2^{x+1})t, where x is the exponent of the terms 1 2 3 4 ... in $S(x)=1^x+2^x+3^x+...$ and $T(x)=1^x-2^x+3^x-4^x+...$ and t is the associated value to T(x).

The number of sliding sums we used is: x+1.

The number of types of slid series to sum is: x+2.

The coefficients correspond to the binomial coefficient '(x+1) choose i', with i that goes from 0 to x+1.

$$\binom{x+1}{i} = \frac{(x+1)!}{i!\,(x+1-i)!}$$

Moreover, it's probably enough to sum until the (x+2)th column to get the result (that's the reason of the tabulation in the table of the previous series).

The result then has to be divided by 2^{x+1} to get t, which is the value of the alternating series T(x). Later, we will have to satisfy the equation: normal → alternating, which is:

$s-2^{x+1}s=t$

hence

$(1-2^{x+1})s=t$

where we will have to insert t to get the value s of the series S(x).

Let's rewrite the previous example, in the light of the new properties:

1+1+1+... = s → 1-1+1-1+...=z (exponent: x=0)

z=s-2s=-s

1 x (1 -1 +1 -1 +1 -...)

1 x (0 +1 -1 +1 -1 +...)

1 +0 +0 +0 +0 +... = 1

$2^{0+1}t=1$ → t=1/2 → s=-1/2

1+2+3+4+...=s → 1-2+3-4+...=z (exponent: x=1)

z=s-4s=-3s

1 x (1 -2 +3	-4 +5 -...)	
2 x (0 +1 -2	+3 -4 +...)	
1 x (0 +0 +1	-2 +3 -...)	
1 +0 +0	+0 +0 +...	

$1=2^{1+1}t$ → t=1/4 → s=-1/12

We avoid to write an all-in-one formula since it will be complicated and not very clear, although it might be full of interesting properties to study.

9) Associated functions

Following a brief explanation of the associated functions which we also see previously.

Only recently I felt the necessity of write this topic, but to explain it in depth I should write many more pages, and the book will become too long and it will be published too late.

We can associate to each series we previously saw a function, which is a “summarizing line of the graph of a series” (we saw this thing in the first chapters of the book)

These functions allow to easily solve complicated case and generalize some concepts. In the 11th chapter we will see some ideas about the “meaning” of all these methods.

I actually don’t know how to completely manage the associated functions.

Moreover, I don’t know if the following methods always coincide with the heuristic results we previously saw, in fact I summarily analysed this new approach, and maybe the following cases are those which are easier or advantageous.

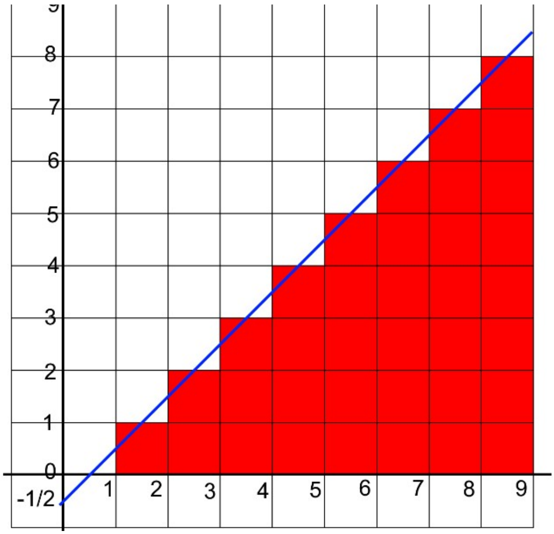
Surely there are some studies of other people about associated series.

For simplicity, the chapter will be divided in paragraphs each one will be dedicated to one, or more of the same type, example(s).

9a) 1+1+1+... and a+a+a+...

In the series 1+1+1+... it’s easy to notice that the line which cuts the histogram is the straight line:

$$1 + 1 + 1 + \cdots = -\frac{1}{2} \quad \rightarrow \quad y = x - \frac{1}{2}$$



in x=0 y is -1/2.

We can generalize it in the following way:

$$a + a + a + \cdots = -\frac{a}{2} \quad \rightarrow \quad y = ax - \frac{a}{2}$$

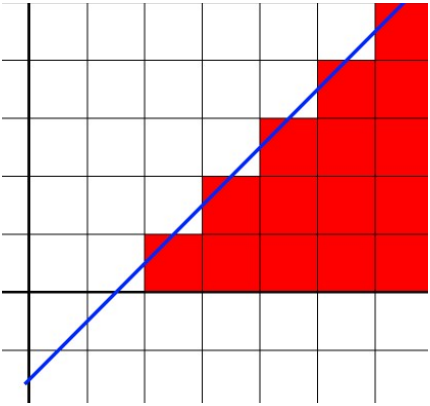
Sliding

We can apply the sliding as a translation of the function.

For the series 0+1+1+1+...=-3/2 (1-sliding) the associated function is:

$$y = x - 1 - \frac{1}{2} = x - \frac{3}{2}$$

$$0 + 1 + 1 + 1 + \cdots = -\frac{3}{2} \quad \rightarrow \quad y = x - \frac{3}{2}$$



which in x=0 has y=-3/2 as we also already saw previously.

We can generalize the sliding in this way:

The associated function to the slid series 0+...+0+1+1+1+... with n 0s at the beginning of the series is:

$$y = x - n - \frac{1}{2}$$

$$0 + 0 + 1 + 1 + 1 + \cdots = -\frac{5}{2} \quad \rightarrow \quad y = x - \frac{5}{2}$$

Add

The translation of the function can be used to calculate an addition to a series, which is the column/vertical sum with other terms (already saw it heuristically in the chapter 5).

Example:

1+1+1+1+1+... = -1/2		+
1+0+0+0+0+... = 1		=
2+1+1+1+1+... = 1/2		

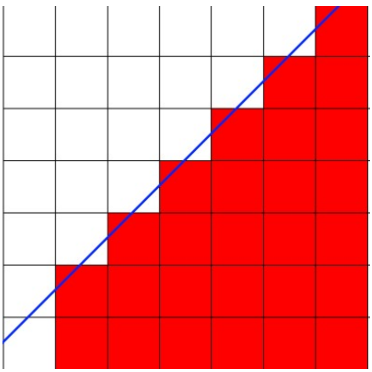
The translation must be done vertically thus the formula is for:

$$a+1+1+1+\cdots=-\frac{1}{2}+(a-1) \quad \rightarrow \quad y=x-\frac{1}{2}+(a-1)$$

For 2+1+1+1+...:

$$2+1+1+1+\cdots=\frac{1}{2} \quad \rightarrow \quad y=x+\frac{1}{2}$$

the histogram graph with the associated function is:



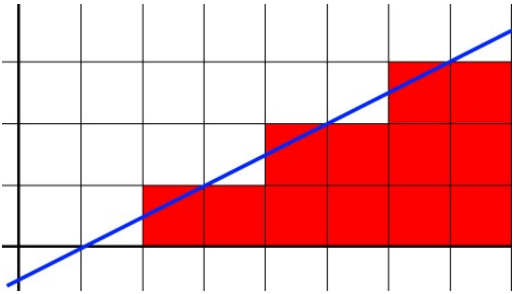
The straight line y=x+1/2 intersects the y axis in 1/2.

Dilatation

The 2-dilation of a series (example: 1+1+1+... → 0+1+0+1+...) can also be applied in the associated function, dilating it of a factor 2.

2-dilation 0x:

$$0+1+0+1+0+1+\cdots \quad \rightarrow \quad y=\frac{x}{2}-\frac{1}{2}$$



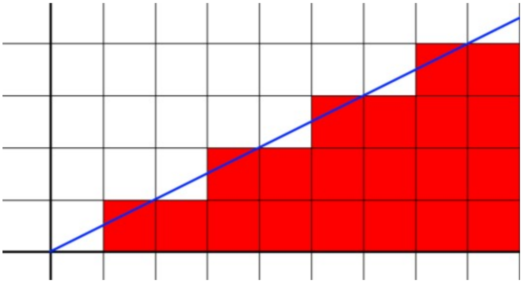
Above, the visualization of the straight line y=x/2-1/2 which intersects the y axis in the ordinate -1/2.

If we want to have the function associated to the series 1+0+1+0+... which is the x0 dilated of 1+1+1+... we just need to translate to left of 1 the function of the 0x dilatated.

$$1+0+1+0+1+0+\cdots \quad \rightarrow \quad y=\frac{x+1}{2}-\frac{1}{2}$$

which is 0 in x=0.

Following the image of the function associated to 1+0+1+0+...



Given a 0...x dilated of the series 1+1+1+..., where the dilation is by a factor d, the associated function is:

$$0+0+\cdots+1+0+0+\cdots+1+\cdots \quad \rightarrow \quad y=\frac{x}{d}-\frac{1}{2}$$

The 0...x dilation (x in the last position) in the heuristic methods, corresponds to a dilation in the associated function.

Whereas the non-0...x dilation in heuristic methods corresponds to a translation of the function associated to the 0...x dilated.

Considering every possible dilation for the series 1+1+1+... the formula becomes;

$$y=\frac{x}{d}+\frac{d-p}{d}-\frac{1}{2}$$

p indicates the position of the term, and d the factor of dilation.

We can notice that the formula we saw in the 6th chapter: (y=-1/2+(d-p)/d) is y=x/d+(d-p)/d-1/2 for x=0, which is the intersection with the y axis.

And a series a+a+a+a+... dilated of d, with a in position p, we have the associated function:

$$y = \frac{ax}{d} + \frac{a(d-p)}{d} - \frac{a}{2}$$

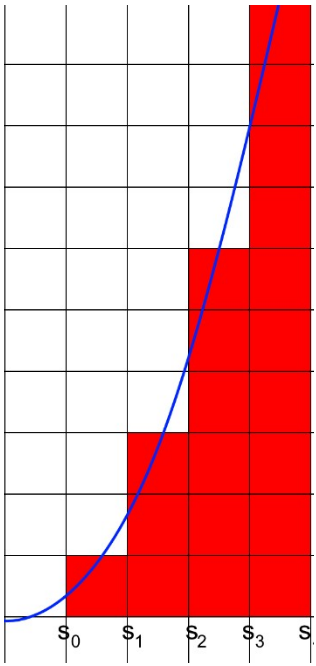
This series has already been shown for x=0 in the 6th chapter.

Combining all these methods we can get a wide range of results.

For example, we can combine a 2-dilation with a 1-sliding and then with another dilation.

9b) 1+2+3+4+...

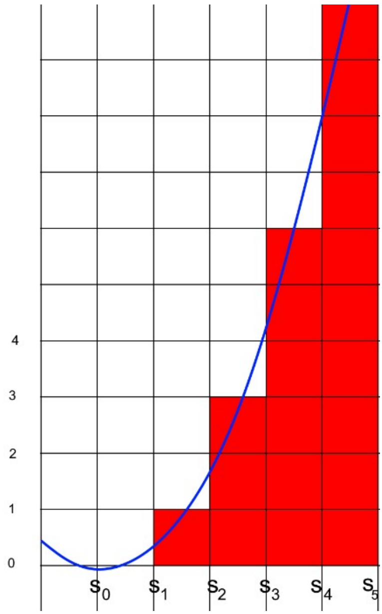
Which is the function of the curve of the graph of 1+2+3+4+...=-1/12?



Remember that the graph is not precise since it’s been done with a simple drawing software and not a math-oriented one.

Sliding

For the series 0+1+2+3+4+...=5/12 we saw a graph similar to the graph of 1+2+3+4+...:



(The graph is non è precise).

the 5/12, we got previously numerically, can be found in the blue curve as an interaction of the blue curve with the y axis.

We already saw the blue curve is a part of a parabola.

Since it’s a parabola, it’s symmetric in relation to the vertical axis which passes in s_0; therefore, the value

5/12 is ordinate also of the intersection of the blue curve with the vertical axis which passes in s_1

We have 3 conditions which we will put in system together with the generic equation of a parabola.

Let's use as symmetry axis the axis passing through s_0:

$$\begin{cases} p: y = ax^2 + bx + c \\ P_1\left(-1, \frac{5}{12}\right) \in p \\ P_2\left(0, -\frac{1}{12}\right) \in p \\ P_3\left(1, \frac{5}{12}\right) \in p \end{cases} \rightarrow \begin{cases} \frac{5}{12} = a(-1)^2 + b(-1) + c \\ -\frac{1}{12} = a0^2 + b0 + c \\ \frac{5}{12} = a1^2 + b1 + c \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} a = \frac{1}{2} \\ b = 0 \\ c = -\frac{1}{12} \end{cases} \rightarrow y = \frac{x^2}{2} - \frac{1}{12}$$

The function associated to 1+2+3+4+... is

$$p: y = \frac{x^2}{2} - \frac{1}{12}$$

If we slide to right of 1 the function, the intersection of the function with the y axis is 5/12 (it's p translated to right of 1):

The function associated to 0+1+2+3+4+... is:

$$y = \frac{(x-1)^2}{2} - \frac{1}{12} = \frac{x^2}{2} - x + \frac{5}{12}$$

Similarly, translating the function of 2 to right, the intersection with the y axis is 23/12, which is the associated value to the 2-slid series: 0+0+1+2+3+4+...=23/12.

The function associated to 0+0+1+2+3+4 is:

$$y = \frac{(x-2)^2}{2} - \frac{1}{12} = \frac{x^2}{2} - 2x + \frac{23}{12}$$

is the function p translated to right of 2.

Add

If we translate the function graph down of 1, we get the intersection is in -13/12. As a matter of fact, -13/12 is the value associated to the series:
0+2+3+4+5+... = -13/12

$$y = \frac{x^2}{2} - \frac{1}{12} - 1$$

this is p translated down of 1.

If we translate the function down of 1 and to left of 1, we get the graph of 2+3+4+5+... which has a intersection with the y axis the ordinate -7/12:

2+3+4+5+...=-7/12

indeed, the function p translated is:

$$y = \frac{(x+1)^2}{2} - \frac{1}{12} - 1$$

Dilatation

We can apply the 2-dilation 0x: we just need to dilate the associated function for the factor 2:

$$0+1+0+2+0+3+0+4+\cdots \rightarrow y=\frac{\left(\frac{x}{2}\right)^2}{2}-\frac{1}{12}=\frac{x^2}{8}-\frac{1}{12}$$

Thus, the intersection of the previous function with the y axis is in -1/12 too.

If we want the function associated to 1+0+2+0+3+0+... we just need to translate to left the function:

$$1+0+2+0+3+0+\cdots \rightarrow y=\frac{(x+1)^2}{8}-\frac{1}{12}=\frac{x^2}{8}+\frac{1}{4}x+\frac{1}{24}$$

which with x=0 is:

$$y=\frac{1}{8}-\frac{1}{12}=\frac{1}{24}$$

It's "confirmed" that 1+0+2+0+3+4+0+...=1/24 (heuristically speaking)

The function of the series 1+2+3+4+... dilated for a factor d is:

$$y=\frac{\left(\frac{x}{d}\right)^2}{2}-\frac{1}{12}=\frac{x^2}{2d^2}-\frac{1}{12}$$

which for d=3 is

$$0+0+1+0+0+2+0+0+3+\cdots \rightarrow y=\frac{x^2}{18}-\frac{1}{12}$$

which for x=0 is again -1/12 (indeed 0+0+1+0+0+2+0+0+3+...=-1/12).

So we can find the associated functions to the dilated series 0+1+0+0+2+0+0+3+0+... and 1+0+0+2+0+0+3+0+0+... translating the function of 0+0+1+0+0+2+0+0+3+... of 1 and 2 to left:

1 to left:

$$0+1+0+0+2+0+0+3+0+\cdots \rightarrow y=\frac{(x+1)^2}{18}-\frac{1}{12}$$

with x=0:

$$y=\frac{1}{18}-\frac{1}{12}=-\frac{1}{36}$$

so

$$0+1+0+0+2+0+0+3+0+\cdots=-\frac{1}{36}$$

2 to left:

$$1+0+0+2+0+0+3+0+0+\cdots \rightarrow y=\frac{(x+2)^2}{18}-\frac{1}{12}$$

with x=0:

$$y=\frac{4}{18}-\frac{1}{12}=\frac{5}{36}$$

so

$$1+0+0+2+0+0+3+0+0+\cdots=\frac{5}{36}$$

<u>Combining all these methods we can get a wide range of results.</u>

For example, we can combine a 2-dilation with a 1-sliding and then with another dilation.

9c) 2+4+8+16+... and geometric series

An unresolved question, in my opinion, is: why geometric series do not change value if slid?

We know that:

2+4+8+16+32+...=-2

0+2+4+8+16+... =-2

0+0+2+4+8+16+...=-2

Usually the sliding in divergent series change the associated value.

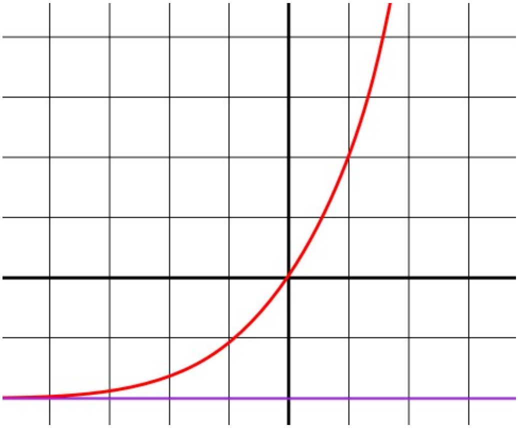
The function of the partial sums of a geometric series is:

$$y = \frac{a - a^{t+1}}{1 - a}$$

with a=2:

$$y = 2^{t+1} - 2$$

its qualitative function graph is:



There is a horizonal asymptote: y=-2

Therefore, a translation to right of the function, will move the intersection of the function with the y axis close to the ordinate -2.

If we think the effective function is similar to this, we can understand why the series 2+4+8+16+... slid gives always -2.

More precisely, slid geometric series give always as value x/(1-x).

Maybe in the associated function the domain is defined only for x>=0 hence -2 is the lower limit.

Or maybe the function change depending on the sliding and the situation is weirder than the previous supposition.

9d) 1+3+5+7+9+...

The function of the partial sums is:

$y = x^2$ which is a parabola

We know that 1+3+5+7+...=1/3

Calculating 0+1+3+5+7+...:

```
+1+3+5+7+9+... = 1/3    +
-2-2-2-2-2-... = 1      +
+1+0+0+0+0+... = 1      =
```

0+1+3+5+7+... = 7/3

Hence, we follow a similar procedure to the one done for the previous case: we put in a system the generic equation of a parabola, and its passage in the point (-1,7/3), (0,1/3), (1,7/3):

$$\begin{cases} p: y = ax^2 + bx + c \\ \left(-1, \frac{7}{3}\right) \in p \\ \left(0, \frac{1}{3}\right) \in p \\ \left(1, \frac{7}{3}\right) \in p \end{cases} \rightarrow \begin{cases} a = 2 \\ b = 0 \\ c = \frac{1}{3} \end{cases} \rightarrow$$

$$1 + 3 + 5 + 7 + \cdots \quad \rightarrow \quad y = 2x^2 + \frac{1}{3}$$

9e) 1+4+9+16+...

Similarly, we can find the function associated to 1+4+9+16+...

The formula of the partial sums is:

$$y = \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{6}$$

Which is not symmetric in the relation to the y axis.

We need to put in a system a generic cubic equation, with the 3 points found from the three sliding applications.

Moreover, it's not symmetric in relation to the y axis so we are not facilitated.

9f) Alternating series

About alternating series, see the method described in the 10th chapter.

9g) Recap table

Following a summarizing table of the functions associated to some of most famous series (including alternating series)

Series	Function of terms	Function of partial sums	Associated function	Associated value
$1+1+1+1+\cdots$	$y=1$	$y=x$	$y=x-\frac{1}{2}$	$-\frac{1}{2}$
$a+a+a+a+\cdots$	$y=a$	$y=ax$	$y=ax-\frac{a}{2}$	$-\frac{a}{2}$
$0+1+1+1+\cdots$	$*_1$	$*_2$	$y=x-\frac{3}{2}$	$-\frac{3}{2}$
$1+2+3+4+\cdots$	$y=x$	$y=\frac{x^2}{2}+\frac{x}{2}$	$y=\frac{x^2}{2}-\frac{1}{12}$	$-\frac{1}{12}$
$0+1+2+3+\cdots$	$y=x$	$y=\frac{x^2}{2}-\frac{x}{2}$	$y=\frac{x^2}{2}-x+\frac{5}{12}$	$\frac{5}{12}$

$2+4+8+16+\cdots$	$y=2^x$	$y=2^{x+1}-2$	?	-2
$1+3+5+7+\cdots$	$y=2x-1$	$y=x^2$	$y=2x^2+\frac{1}{3}$	$\frac{1}{3}$
$0+1+0+1+\cdots$	$*_4$	$*_4$	$y=\frac{x}{2}-\frac{1}{2}$	$-\frac{1}{2}$
$1+0+1+0+\cdots$	$*_4$	$*_4$	$y=\frac{x}{2}+\frac{1}{2}$	0
$0+1+0+2+$ $+0+3+0+4+\cdots$	$*_4$	$*_4$	$y=\frac{x^2}{8}-\frac{1}{12}$	$-\frac{1}{12}$
$1-1+1-1+\cdots$	$y=1$ $*ALT.$	$*_3$	$y=\frac{1}{2}$	$\frac{1}{2}$
$1-2+3-4+\cdots$	$y=x$ $*ALT.$	$*_3$	$y=\frac{1}{4}$	$\frac{1}{4}$

$*_1$ It’s a piecewise function, starts as y=0, then from x=1 it’s y=1 (potential particular case)

$*_2$ It’s a piecewise function, starts as y=0, then from x=1 it’s y=x (potential particular case)

$*_3$ It’s a piecewise function (or in points), because the series is alternating, for a more in-depth explanation see dedicated paragraph.

*ALT. This is a function of the absolute values of the terms, since they are alternating.

*$_4$ For brevity, we omit the analysis of the function, which is piecewise but can be reconducted to a continuous function.

Note: x is a natural number, and we usually start with the index 1 (index 1: first term) usually it’s convention to make the first index 0 to indicate the first term.

At the moment I don’t know a general function associated to the patterned series. They are very important though. At the web page “bookerrors” (which link is in the beginning and at the end of the book) will be written if available.

The combination of the methods previously described can give a wide range of methods to solve manty series.

10) Graphs, stability detection and value extraction

10a) Line graph vs histogram graph

In the book are shown line graphs and histogram graphs, both about the partial sums of infinite series.

Remember that the line graph is not a good instrument to analyse a series, while the histogram is good.

The line graph (with the associated function plotted) often shows results which are not coherent with numerical calculation, while the histogram graph (with the associated function plotted) shows results coherent with the values numerically found.

Following a summarizing table of some cases:

	Line	Histogr.	Numeric.
1-1+1-1+...	1/2	1/2	1/2
1-2+3-4+...	1/4	1/4	1/4
1+1+1+1+...	0	-1/2	-1/2
1+2+3+4+...	?	-1/12	-1/12
1-2+1-2+...	0	1	1

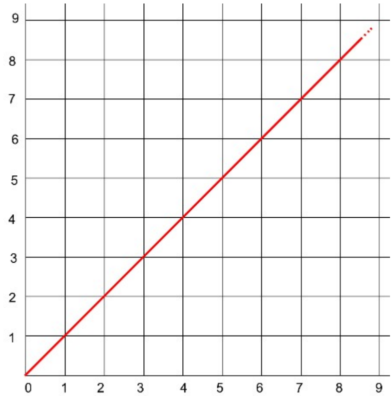
The line graph is easier to draw and analyse though.

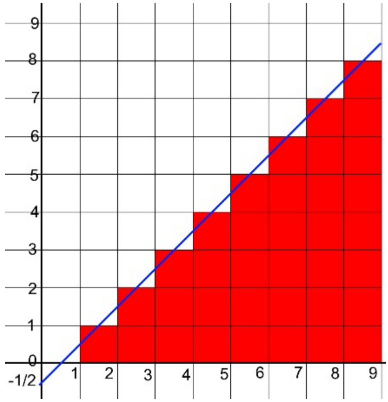
In stable series, the line graph (with the associated function plotted) brings to the same results of the histogram (with the relative associate function plotted).

An important problem of the line graph is that the transition of a partial sum to its consecutive is progressive, while in fact it’s instantaneous.

Let’s see for example the case of 1+1+1+1+...

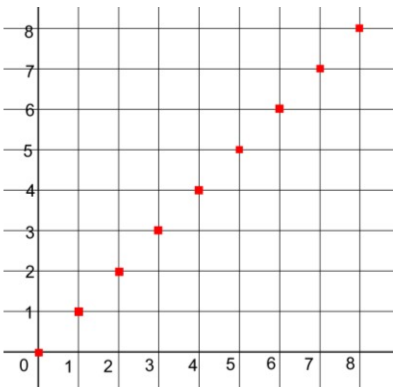
Comparing the line graph with the histogram graph:





In every term sum, a 1 is added, but only in the histogram the instantaneous sum is showed.

We can also find another interesting type of graph:

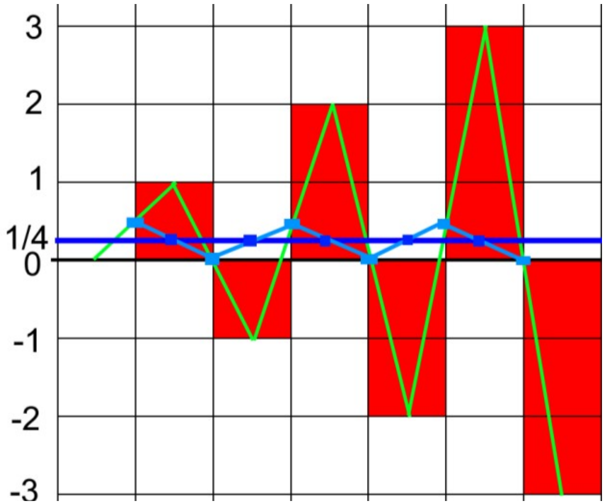


This case includes only the points relative to the natural numbers (partial sums indexes/numeration).

The line graph is a connection of all points of the points graph (scatter plot).

10b) Stability detection and value extraction

We already saw in the previous chapter that the histogram graph of 1-2+3-4+5-6+... is:



We can notice that in a histogram graph of a series, we can check if the series is stable through the following algorithm:

1) Mark the midpoints of each horizontal side of every column

(In the previous image, the midpoints are the vertexes of green polygonal chain)

2) Draw the polygonal chain which connects the midpoints.

(In the previous image, it’s the green polygonal chain)

3) Mark the midpoints of the (new) segments.

(In the previous image, they are the light blue markers)

4) Go to instruction 2

Continue until the points lay on a straight line.

If the straight line is parallel to the x axis, then the series is stable;

else it’s not.

The intersection of the straight line (regardless it’s parallel or not to the x axis) with the y axis gives as ordinate the value associated to the series.

If no straight line is generated after a number of iterations, then the algorithm is not appropriate to extract the value of the series.

Therefore, the algorithm can be used to detect if a series is stable, but it can extract the value only if a straight line is generated (hence, it can’t be used in divergent series with non-constant terms)

At the moment, I don’t know any algorithm for divergent series with non-constant terms.

Notice how the first green polygonal chain coincides with the line graph (of the same series) slid to right of 1/2.

Therefore, the algorithm can be applied to the histogram, starting from the instruction 1, or to the line graph slid of 1/2 to right, starting from the instruction 3.

The algorithm can be managed also only through numbers (considering the coordinates of the points of the graph) but it will be complicated and long to explain, so the explanation is omitted for brevity.

Just a hint though: the binomial coefficient appears indirectly.

It’s possible the formula will be very useful.

Line graphs are much easier for verifying the stability of a series, but not for extracting its value.

The procedure of value extraction for series where the algorithm does not work is described in the 9th chapter.

11) Model and properties

New properties can be seen in the infinite series.

11a) Commutative property

The commutative property does not change the value of a non-convergent series only if it's applied to a non-infinite quantity of terms.

Example:

+1-1+1-1+1-1+... = 1/2

Swapping the first term with the second:

-1+1+1-1+1-1+... = -1+1+1/2 (stability)

-1+1+1/2 = 1/2

Coherent.

If we commute all the +1s with the consecutive -1s (infinite quantity of terms) the series changes value:

-1+1-1+1-1+1-...=-1/2

Example:

1+2+1+2+1+2+...=-1

and also 2+1+1+2+1+2+... = -1

if we commute every term of odd positions with the consecutive term, we get:

2+1+2+1+2+1+... = -1/2

In the convergent series the commutation of an infinite quantity of terms does not change the associated value.

11b) Associative property and grouping

The associative property can be applied only to convergent or stable or geometric series, if not the value of the series will change.

We already studied the problem a little when we faced the parenthesis problem.

E.g.: (divergent)

1+1+1+1+1+...= -1/2

but

(1+1)+1+1+... → 2+1+1+1+... = 1/2

It’s possible to apply the associative property in alternating stable series:

In the light of the consideration that the vertical approach is right (while the horizontal approach is generally not valid) for the series sum, we need to establish a convention for the parenthesis.

If I write: 1+(1+1+1+...) in the parenthesis is the series 1+1+1+... or it minus 1 (which is 0+1+1+1+...)?

Let’s establish the following convention:

if we write 1+(1+1+1+...) we mean: 1 plus the series 1+1+1+1+... thus the series 1+1+1+... is "complete", even if there is another 1 out of the parenthesis.

If we want to highlight the lack of a 1, we must write: 0+1+1+1+... meaning we have to subtract a 1 to the first term.

Therefore, the grouping in the example:

1+1+1+1+.... = 1+(1+1+1+1+...) NOT CORRECT!

since in the parenthesis we should have write (0+1+1+1+...), better if in a vertical writing:

```
1+1+1+1+... = 1+0+0+0+...    +
              0+1+1+1+...    =
---------------------------------
1+1+1+1+... = 1+1+1+1+...
```

Grouping in non-stable or non-convergent series, is allowed only if non-partial (which means only if something is gathered from all terms and not from part of them, if the latter, associative property would be applied)

In stable series associative property and partial grouping is allowed.

Exception!

Geometric series allow the application of the associative property and partial grouping

I suppose geometric series and stable alternating series are intimately connected, since both of the two types allow associative properties.

Associative property is connected to the horizontal approach.

11c) “Futility”

In some cases, part of a series seems to “not have any effect in the result”:

500+1+500+1+500+1+...=-1/2

The terms 500 do not have any effect:

500+0+500+0+500+0+...=0 +

 0+1+ 0+1+ 0+1+...=-1/2 =

500+1+500+1+500+1+...=-1/2

Remember that the series 500+0+500+0+... has value 0 because it’s a x0 dilated series.

11d) The = problem

The problem of the ‘=’ starts when a series is not convergent and is associated to a value: putting a convergent series like 1/2+1/4+1/8+... = 1 is ok, as a matter of fact, the sequence of the partial sums partial tends to 1; whereas mathematically it’s not correct to say that 1-1+1-1+...=1/2 or that

1+2+3+4+...=-1/12, since these two series do not converge! their sequence of partial sums do not tend to those values.

First idea

The value associated to non-convergent series is a “reflection” of what happens in its associated function (summarizing line) which intersects the y axis in the histogram graph of its partial sums and gives as ordinate that value.

We already saw associated functions in the 9th and 10th chapters, and many other times.

In my opinion, however, this does not explain completely the problem, I mean: why and how this association happens? Why and how heuristic methods give the same value of the ordinate of the intersection of the associated function with the y axis? What’s the meaning of this?

Is this all caused by the use of an infinite quantity of terms?

An alternative idea can be useful.

Second idea

As already said by other intellectuals, we can extend the concept of sum.

In particular, we can notice, in the light of the new things explained before (grouping, histogram graphs, associated functions, etc.), that the terms and their sums do not have just a quantitative meaning: the disposition of the terms is fundamental.

About the series 1-1+1-1+... (and the relative lamp anecdote) we can notice that the terms and relative sums are temporal.

We also can think that the sum, in the “infinite” context, loses its non-temporal characteristic, or maybe the sum it’s just not a “classic” sum.

We can also find a “dimensional-change”: usually a normal sum acts based on quantity, which can easily be represented on one line (like the axis). When we add infinites, the use of the y axis (new dimension) is necessary to describe the results (as the x axis is occupied from the partial sums indexes)

This is easily associable to the duo: horizontal approach (x axis) which is typical of non-infinite calculations, and vertical approach (y axis) which is typical of many infinite series.

A “dimensional change” can be seen in the following situation:

+2+2+2+2+... = -1

becomes

+2+2+2+2+... = 1-1

+1+0+0+0+...

————————————

3+2+2+2+... = 0

Moving a term from a finite member to an infinite member implies a sign change and a dimensional change too (from horizontal sum to vertical sum).

But it’s true that the horizontal sum does not exclude the vertical sum.

This is rather an idea than an affirmation, since many inconsistent or unclear things can be found.

Know that a term a can be saw as the infinite series a+0+0+0+...

Moreover, we can ask ourselves if the z axis can be used for somethings similar.

I know the previous paragraph is very vague, but the question is still unclear to me too.

Moreover, we can also notice how the idea of 1-1+1-1+... which becomes 1/2 can be associated also to non-temporality (a point of view totally opposite to the previous one); as a matter of fact we saw in a paragraph of the 1st chapter (“famous series”) that 1/2 can be interpreted as the value of 1-1+1-1+... from a reference system “out of time”. When the value is associated, the time is not considered, whereas 1-1+1-1+... behave as temporal since every term is periodically summed.

Detemporalization is “seeing from a non-temporal point of view” “from out of the time dimension”, looking from outside in, also, “looking from a upper level” (like an airplane see the whole aspect of a territory which would be impossible or very difficult from someone on the ground).

Thus, we can introduce a new paragraph: levels.

I know, these paragraphs are quite non-rigorous and unclear.

11e) Levels

In the light of the new ideas we did, we can think that in the series 1-1+1-1+... the result 1/2 can be localized in a "upper level" which now we will indicate as l1. The terms 1 and -1 are instead in the level 0.

Similarly, for the series 1+1+1+1+...=-1/2:

liv. 1:   -1/2

liv. 0:   1+1+1+1+... = +∞

In this case the +∞ of 1+1+1+... can be "read" as -1/2 in the upper level, and maybe we can think that the -1/2 is a reflection of 1+1+1+... and not of +∞ which actually already lost the information about the -1/2.

We can also think many other levels are above l1.

We can think, all these levels are "levels of infinity"

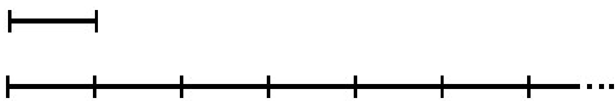
And we can connect to geometry, in the topic of infinity (half-line, line, space, space, point, ... they all use the concept of infinity).

11f) Geometry

The 1+1+1+1+... can be connected to the idea of concatenation of segments, each one of unitary length.

An infinite quantity of concatenated segments in the same verse gives a half-line.

segment of length 1



half-line

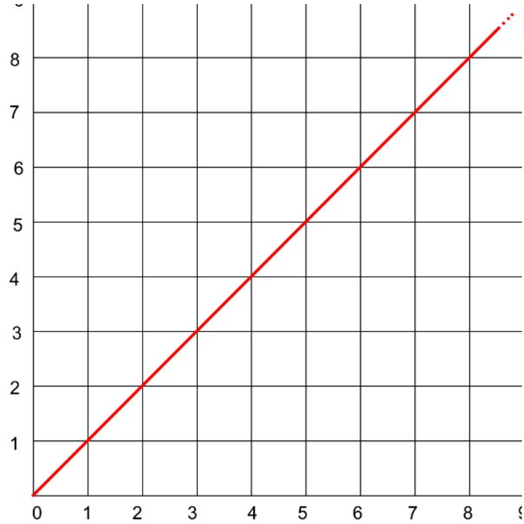
We can suppose that the half-line is the main one-dimensional entity instead of the line, which becomes two times the half-line (usually line is the main one-dimensional entity, and the half-line is indeed half of it).

This is an idea, and not an affirmation.

But we saw that the association of a value to a divergent infinite series depends also from how the divergence is reached. For example, both 1+1+1+... and 1+2+3+4+... diverge, but they have different value, respectively -1/2 e -1/12.

Thus, in the geometric comparison we have to evaluate also every term.

We can do it using the cartesian plan and a graph.



The x axis indicates the number of terms added, and the y axis indicates the x-th partial sum.

But we saw that the line graph is not good to extract the value of a series, whereas the histogram graph is good to do it.

However, in this chapter we will use the line graph to make things simple, which can easily be reconducted to the histogram. <u>The topic however should be further explained also through histograms.</u>

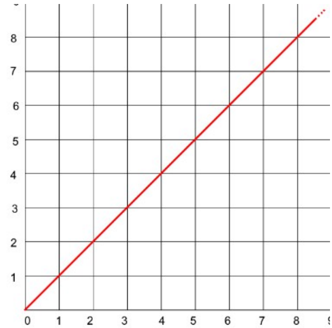
We also know that 1+1+1+1+...=-1/2

How can we represent this result geometrically?

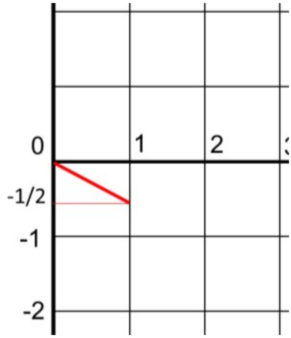
Thinking about the idea of more levels previously explained, we can make an association between the half-line y=x in the quadrant I of the graph with the vector

v = (1 , -1/2)

that means:



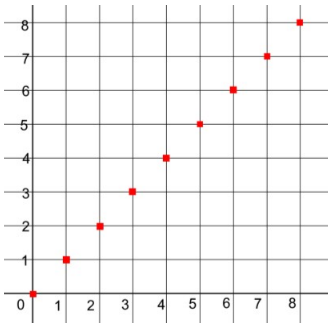
→



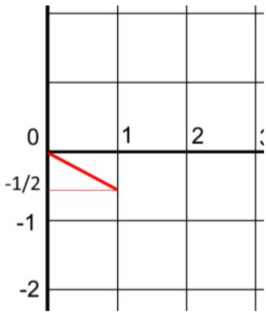
at level 0 at level 1

This idea can be furtherly developed and bring new ideas.

Like for example the fact that maybe is more correct associate the point graph to the vector v



→



at level 0 at level 1

Maybe we are dealing with a points “condensation”.

But this is not working much with the function which are not straight lines (like the function of 1+2+3+4+... which is $y=x^2/2+x/2$)

Going back to the question line/half-line, which is the main entity?:

The line would be associated to the double verse series:

s: ... +1 +1 +1 +1 +1 +...

More in depth studies about this topic connect to a different way of thinking how numbers are “organized”: non-standard analysis is a good idea but even more interesting concepts can be studied. However, this topic is omitted for brevity.

12) Open problems

Many times, in this book have been reported, via underlining, many problems still unclear to me or unresolved by me, which deserve to be studied better.

I specify the "me", since maybe they have already been solved and it's me who doesn't know.

Following a list of some of the open problems:

- Break of the behaviour of the formula

  $$x + x^2 + x^3 + x^4 + \cdots = \frac{x}{1-x}$$

  in x=1, where it's not defined, but

  $$1+1+1+1+... = -1/2$$

  And presence of asymmetry:

  $$\frac{\lim_{x\to 1^+} \frac{x}{1-x} + \lim_{x\to 1^-} \frac{x}{1-x}}{2} = -1$$

  while the point is in the ordinate -1/2 (1+1+1+...= -1/2).

  Is there an asymmetry in the "structure" of the infinities?

- non 0...0x dilation formula for divergent series with non-constant terms and not patterned, with heuristic methods of direct manipulation of series (which means not using associated functions)

- A method to find the summarizing line/curve in a series with non-constant terms and not patterned.

- Extension of the formula

$$s = \frac{1}{2}\sum_{i=1}^{n} x_i - \frac{1}{n}\sum_{i=1}^{n} x_i i$$

for n tending to infinity.
It does not work, why?

For example: the series 1+2+3+4+... can be thought as a patterned series, with an infinite length pattern of elements: 1, 2, 3, 4, ...

Hence the formula becomes:

$$s = \lim_{n\to+\infty}\left(\frac{1}{2}\sum_{i=1}^{n} x_i - \frac{1}{n}\sum_{i=1}^{n} x_i i\right) =$$
$$= \lim_{n\to+\infty}\left(\frac{1}{2}\left(\frac{n^2}{2}+\frac{n}{2}\right) - \frac{1}{n}\left(\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}\right)\right) =$$
$$= \lim_{n\to+\infty}\left(\frac{n^2}{4}+\frac{n}{4}-\frac{n^2}{3}-\frac{n}{2}-\frac{1}{6}\right) = -\infty$$

which does not give -1/12

The fact that the coincide does not occur is quite obvious, but further explanation would be interesting.

- Double verse infinite series:
  ... +1 +1 +1 +...

- Bidimensional expansion:

An interesting problem comes when an infinite series is considered as the vertical sum of more infinite series.
1+2+3+4+... for example can be:

```
1 +1 +1 +1 +1 +... = -1/2 -0
0 +1 +1 +1 +1 +... = -1/2 -1
0 +0 +1 +1 +1 +... = -1/2 -2
0 +0 +0 +1 +1 +... = -1/2 -3
0 +0 +0 +0 +1 +... = -1/2 -4
⋮  ⋮  ⋮  ⋮  ⋮
=  =  =  =  =
1 +2 +3 +4 +5 +...
```

Why the sums in the two different dimensions are not equal?:

(-1/2-0)+(-1/2-1)+(-1/2-2)+(-1/2-3)+... =
= -1/2-3/2-5/2-7/2-... =
= -1/2(1+3+5+7+...) = -1/2 * 1/3 = -1/6

while 1+2+3+4+...=-1/12.
The problem must be studied and solved.

A curious case is:

```
 +1 +1 +1 +1 +... = -1/2
 +1 +1 +1 +1 +... = -1/2
 +1 +1 +1 +1 +... = -1/2
 +1 +1 +1 +1 +... = -1/2
  ⋮  ⋮  ⋮  ⋮
```
$$-\frac{1}{2}-\frac{1}{2}-\frac{1}{2}-\frac{1}{2}$$

And what is its meaning?

- Series with complex terms:
  Example:

  1+1i + 1+2i + 1+3i + 1+4i +... =? -1/2-i/12

- Extension of methods for non-natural factors and their eventual representation in series:
  for example:
  while the dilation for a factor 2 of 1+1+1+1+... corresponds to 0+1+0+1+... or 1+0+1+0+..., what would correspond the series 1.5 dilated?

  Similarly, for sliding or other.

- Methods of direct manipulation of series to get the series 0+1+0+0+2+0+0+3+0+... and 1+0+0+2+0+0+3+0+0+...

  Maybe the general method to find 'non 0...x dilated series of a series with non-constant terms and not patterned', uses the patterned series and/or alternating series, or maybe the bidimensional expansion.

- Three-dimensional or n-dimensional expansions of a series and its meaning.

13) Infinite series list with their values

1-1+1-1+... = 1/2

1-2+3-4+... = 1/4

1-4+9-16+... = 0

1-3+5-7+9-11+... = 0

$x+x^2+x^3+x^4+... = x/(1-x)$

$x-x^2+x^3-x^4+... = x/(1+x)$

$1+x+x^2+x^3+... = 1/(1-x)$

$1-x+x^2-x^3+... = 1/(1+x)$

1+1+1+1+... = -1/2

x+x+x+x+... = -x/2

n+a+a+a+... = -a/2 + (n-a)

a+...+a+n+a+a+... = -a/2 + (n-a)

1+2+3+4+... = -1/12

2+4+6+8+... = -1/6

1+4+9+16+... = 0

0+1+1+1+... = -3/2

0+0+1+1+1+... = -5/2

2+1+1+1+1+... = 1/2

0+1+2+3+4+... = 5/12

0+0+1+2+3+4+... = 23/12

2+3+4+5+... = -7/12

0+2+3+4+... = -13/12

0+1+0+1+0+1+... = -1/2

0+x+0+x+0+x+... = -x/2

1+0+1+0+1+0+... = 0

x+0+x+0+x+0+... = 0

0+0+1+0+0+1+... = -1/2

0+1+0+0+1+0+... = -1/6

1+0+0+1+0+0+... = 1/6

General formula of dilatation of a series a+a+a+...

D(a,d,p) = -a/2 + a(d-p)/d

0+1+0+2+0+3+... = -1/12

$0+x_1+0+x_2+0+x_3+...$ = value di $x_1+x_2+x_3+...$

1+0+2+0+3+0+... = 1/24

1+0+0+2+0+0+3+0+0+4+0+0+... = 5/36

0+1+0+0+2+0+0+3+0+0+4+0+... = -1/36

0+0+1+0+0+2+0+0+3+0+0+4+... = -1/12

1+3+5+7+... = 1/3

0+1+3+5+7+... = 7/3

0+1+0+3+0+5+... = 1/3

1+0+3+0+5+0+… = 1/12

2+4+6+8+... = -1/6

1-2+1-2+1-2+... = 1

a+b+a+b+a+b+... = -b/2

1+1+2+2+3+3+... = -1/24

1-1+2-2+3-3+... = 1/8

1+2+3+1+2+3+... = -5/3

a+b+c+a+b+c+... = (a-b-3c)/6

a+b+c+...+n+a+b+c+...+n+...=

$$s = \frac{1}{2}\sum_{i=1}^{n} x_i - \frac{1}{n}\sum_{i=1}^{n} x_i i$$

where x_i is the i-th term of the pattern, and N is the number of components of a pattern

Errata corrige del 6 giugno 2020

Questa errata corrige potrebbe a sua volta contenere errori.

Dove non vengono indicati alcuni aspetti (come ad esempio il formato del testo, o le evidenziazioni) si intende che essi rimangono invariati.

Le sottolineature indicano che il testo doveva essere riportato sottolineato (indicazione di problema aperto o irrisolto).

Il testo in corsivo indica invece una nota, che non deve però appartenere al libro: è una nota di chiarimento sulla correzione.

Pag.	Versione stampata	Versione corretta
31	Invece, il grafico a istogramma interseca a -1/2	Invece, il grafico a istogramma porta indirettamente al valore a -1/2
31	La dimostrazione è simile a quella di Ramanujan	La “dimostrazione” è simile a quella di Ramanujan
35	Al momento non mi è chiaro il metodo da usare per generare la curva blu (il metodo usato per 1+1+1+1+... i per ...	Al momento non mi è chiaro il metodo da usare per generare la curva blu in tutti i casi simili a questo... (il metodo usato per 1+1+1+1+... e per ...
41	A’’;	A’’:
41	Il valore di A’ è uguale a A poiché A è stabile	Il valore di A’ è uguale al valore di A poiché A è stabile
55	Non ho verificato se è possibile applicare la compressione alle serie geometriche.	<u>Non ho verificato se è possibile applicare la compressione alle serie geometriche con successo.</u> *La frase doveva dunque essere sottolineata in quanto delinea un problema aperto.*
58	U è la up compressione, D è la down compressione	se attuata una 2-compressione, U è la up compressa, D è la down compressa
59	La linea viola superiore (nel primo quadretto coincide con il grafico rosso) è la linea rappresentate la Up serie ...	La linea viola superiore (nel primo quadretto coincide con il grafico rosso) è la linea rappresentante la Up serie ...
60	... costanti e non a pattern con risultati efficienti e sensati ...	... costanti e non a pattern con risultati efficienti e sensati, anche se è probabile che la 2-compressione possa essere applicata con successo alle serie geometriche divergenti.
63	costanti e non a pattern con risultati efficienti e sensati.	costanti e non a pattern con risultati efficienti e sensati, anche se è probabile che la 3-compressione possa essere applicata con successo alle serie geometriche divergenti.

63	Non possibile.	Non possibile. anche se è probabile che una n-compressione possa essere applicata con successo alle serie geometriche divergenti.
70	Quando dà -2+3-4+...=?	Quanto dà -2+3-4+...=?
77	Quanto dà 2+3+4+...=7/12	Possiamo trovare che: 2+3+4+...=-7/12
78	Se andiamo ad osservare la funzione associata alle somme parziali e la linea ...	Se andiamo ad osservare la funzione delle somme parziali e la linea ...
83	Una dilatazione è per un fattore n positivo intero.	Una dilatazione è per un fattore n positivo intero, che indicheremo con n-dilatazione.
83	La 2 dilatazione è molto importante.	La 2-dilatazione è molto importante.
86	Notiamo che la serie 0+1+0+1+0+1+... è la serie 1+0+1+0+... slittata di -1. E in modo analogo per le serie 0+a+0+a+... di a+0+a+0+...	Notiamo che la serie 0+1+0+1+0+1+... è la serie 1+0+1+0+... slittata di 1. E in modo analogo una serie 0+a+0+a+... è la 1-slittata di a+0+a+0+...
87	Notiamo che una serie 0+1+0+2+0+3+... è la serie 1+0+2+0+3+0+... slittata di -1. E in generale per una serie $0+x_1+0+x_2+...$ di $x_1+0+x_2+0+...$	Notiamo che la serie 0+1+0+2+0+3+... è la serie 1+0+2+0+3+0+... slittata di 1. E, in generale, una serie $0+x_1+0+x_2+...$ è la 1-slittata di $x_1+0+x_2+0+...$
92	... a partire da una serie divergente a termini non costanti con la manipolazione ...	... a partire da una serie divergente a termini non costanti e non a pattern, con la manipolazione...
93	Essendo la 3-dilatazione, una dilatazione per un fattore 3, vi saranno 3 serie intermedie principali:	Essendo la 3-dilatazione una dilatazione per fattore 3, vi saranno 3 tipi principali di 3-dilatazione:
94	La linea blu interseca l'asse y a 1/2 come la serie 1+1+1+1+...=-1/2	La linea blu interseca l'asse y a 1/2 come la serie 1-1+1-1+...=1/2
95	La 3-dilatazione 00x non cambia il valore di una serie a termini non costanti	La 3-dilatazione 00x non cambia il valore di una serie divergente a termini non costanti e non a pattern
97	È necessario usufruire delle funzioni associate (vedi capitolo 10:	È necessario usufruire delle funzioni associate (vedi capitolo 9):
98	La 3-dilatazione è analoga simile alla 2-dilatazione paritaria ...	La 3-dilatazione è simile alla 2-dilatazione paritaria (solo che per un fattore 3, invece che 2)
98		*All'inizio del capitolo 6c da aggiungere:*

		Per una n-dilatazione esistono n tipi di n-dilatazioni possibili principali (come per la 3-dilatazione erano possibili 3 tipi principali di 3-dilatazione: 00x, 0x0, x00) 0....0x 0...0x0 ... 0..x..0 ... 0x0...0 x0....0 Per comodità (senza bisogno di fare calcoli ogni volta) ricordiamoci che: Una n-dilatazione del tipo 0...x non cambia il valore ad una serie. (ricordiamoci infatti che già le dilatazioni 0x, oppure 00x non cambiavano i valori di una serie; in modo analogo per tutte le n-dilatazioni del tipo 0...x).
102	Per trovare il nome della dilatazione finale, bisogna dunque applicare le dilatazioni ai termini di dilatazioni.	Per trovare il nome della dilatazione finale, bisogna dunque applicare la seconda dilatazione al nome della prima dilatazione.
102	-2-4-6-8-12-... = -2(1+2+3+..) =1/6	-2-4-6-8-12-... = -2(1+2+3+...)=1/6
115	4^x+... e ti è il valore associato a T(x)	4^x+... e t è il valore associato a T(x)
116		*A fine capitolo era da aggiungere:* Per quanto riguarda le funzioni associate (vedi capitolo 9): interessante sarebbe trovare eventuali funzioni associate alle serie a pattern, che per ora non ho però ricercato.
120	1+1+1+1+1+... = -1/2 1+0+0+0+0+... = 1 2+1+1+1+1+... = 1/2	1+1+1+1+1+... = -1/2 + 1+0+0+0+0+... = 1 = 2+1+1+1+1+... = 1/2
123		*La pagina presenta vari paragrafi appartenenti ad un'altra sezione. La pagina 123 è da eliminare tutta, tranne l'ultimo riquadro, che è corretto e al posto giusto:* *"E per una serie a+a+a+a+..."*
129	Una questione aperta è: perché le serie geometriche slittate non cambiano valore?	Una questione aperta è: perché le serie geometriche slittate non cambiano valore come invece fanno

		le serie divergenti non a pattern ma non geometriche? *La frase era da riportare sottolineata in quanto descrive un problema aperto.*
133		*Sulla riga relativa alla serie 1+3+5+7+... la funzione associata ha un'evidenziazione per un errore di stampa*
133	*Sulla riga relativa alla funzione 0+1+0+2+0+3+0+4+... la funzione associata è:* $\frac{x^2}{8}-\frac{1}{12}$	*Versione corretta:* $y=\frac{x^2}{8}-\frac{1}{12}$
134	*ALT. La funzione è quella riportata ma in versione alternata ossia se è riportata la funzione.	*ALT. La funzione indica l'andamento dei valori assoluti dei termini (per x=1,2,3,... con $x \in \mathbb{N}-\{0\}$). I termini nella serie sono in realtà a segno alterno (+,-,+,-,...).
134	*$_4$ Per brevità omettiamo l'analisi della funzione dei termini, che è a tratti ma riconducibile ad una continua...	*$_4$ Per brevità omettiamo l'analisi della funzione, che è a tratti ma riconducibile ad una continua...
134		*Da aggiungere a fine capitolo* Interessante sarebbe studiare eventuali funzioni associate alle serie a pattern.
136	In questo caso si limita la funzione delle somme parziali ai soli numeri naturali, e il grafico a linee...	In questo caso si visualizza la funzione delle somme parziali solo per i numeri naturali (che sull'asse x rappresentano gli indici delle varie somme parziali), e il grafico a linee...
138	L'intersezione della retta (indipendentemente ... ... il valore associato alla serie.	*Da riportare evidenziato in un riquadro arancione.*
141	La proprietà associativa non può essere ...	La proprietà associativa non può essere applicata alle serie non convergenti, né alle serie non stabili, né a quelle non geometriche; altrimenti il loro valore cambierebbe.
155		*Aggiungere un punto alla lista dei problemi aperti:* • Studiare e verificare se è possibile applicare con successo la compressione alle

			serie geometriche (incluse anche quelle alternate con i valori assoluti dei termini in andamento geometrico) • perché le serie geometriche slittate non cambiano valore come invece fanno le serie divergenti non a pattern ma non geometriche? • Funzioni associate alle serie a pattern

CONTACTS

writer of the book (me writing now):
andrea sorato

telegram
@rifmod

website
rifmod.com